7. [11 points] Let g be a differentiable function defined for all real numbers satisfying all of the following properties:

- $g(5) = 4$.
- $g(x)$ has a local maximum at $x = -2$ and $g(-2) = 3$.
- $g(x)$ has a local minimum at $x = 1$ and $g(1) = -1$.
- g has exactly two critical points.
- $\lim_{x\to\infty} g(x) = +\infty$.
- $\lim_{x\to-\infty} g(x) = 0$.

a. [3 points] Circle all of the following intervals on which $g'(x)$ must be always positive.

$\boxed{x < -2}$ $\quad -2 < x < -1 \quad -1 < x < 1 \quad$ $\boxed{1 < x < 3}$ $\quad$ $\boxed{3 < x < 5}$ $\quad$ $\boxed{5 < x}$

b. [4 points] Find all the values of x at which $g(x)$ attains global extrema on $-2 \le x \le 5$. If not enough information is provided, write NOT ENOUGH INFO. If there are no such values of x, write NONE. Briefly indicate your reasoning.

Solution: Since $g(x)$ has local extrema at $x = -2$ and $x = 1$, $g(x)$ has critical points at these values. As stated in the problem, g has exactly 2 critical points. Thus, these are the only critical points.
Since $g(x)$ is continuous and $-2 \le x \le 5$ is a closed interval, $g(x)$ attains global extrema on this interval, and they occur at critical points or end points. By comparing the values of $g(x)$ at the critical points and endpoints ($g(-2) = 3$, $g(1) = -1$, and $g(5) = 4$), we conclude the following:

Answer: global min(s) at $x =$ 1

Answer: global max(es) at $x =$ 5

c. [4 points] Find all the values of x at which $g(x)$ attains global extrema on its domain. If not enough information is provided, write NOT ENOUGH INFO. If there are no such values of x, write NONE. Briefly indicate your reasoning.

Solution: Since $\lim_{x\to\infty} g(x) = +\infty$, g has no global maximum on its domain.

We claim that the function $g(x)$ has a global minimum value of -1 at $x = 1$. To see this, first note that $g(x)$ is increasing for $x > 1$. So $g(x) > g(1) = -1$ for $x > 1$. Now, $g(x)$ is decreasing for $-2 < x < 1$, so $g(x) > g(1) = -1$ for $-2 < x < 1$. Finally, $g(x)$ is increasing for $x < -2$ and $\lim_{x\to\infty} = 0$, so $g(x) > -1$ for $x < -2$. (In fact, $g(x) \ge 0$ for $x < -2$.)
Combining this information, we conclude that $g(1) = -1$ is the minimum value achieved by $g(x)$ on its domain.

Answer: global min(s) at $x =$ 1

Answer: global max(es) at $x =$ NONE