9. [10 points] Our friend Oren, the Math 115 student, wants to minimize how long it will take him to complete his upcoming web homework assignment. Before starting the assignment, he buys a cup of tea containing 55 milligrams of caffeine.
Let $H(x)$ be the number of minutes it will take Oren to complete tonight's assignment if he consumes x milligrams of caffeine. For $10 \leq x \leq 55$

$$H(x) = \frac{1}{120}x^2 - \frac{4}{3}x + 20\ln(x).$$

Instead of immediately starting the assignment, he solves a calculus problem to determine how much caffeine he should consume.

a. [8 points] Find all the values of x at which $H(x)$ attains global extrema on the interval $10 \leq x \leq 55$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema.

Solution: Since $H(x)$ is continuous on the interval $10 \leq x \leq 55$, by the Extreme Value Theorem, $H(x)$ attains both a global minimum and a global maximum on this interval. These will occur at either endpoints or critical points.
Now,

$$H'(x) = \frac{x}{60} - \frac{4}{3} + \frac{20}{x} = \frac{x^2 - 80x + 1200}{60x} = \frac{(x-60)(x-20)}{60x}.$$

Thus, $H(x)$ has exactly one critical point on the interval $10 \leq x \leq 55$, and it is at $x = 20$. To determine the global extrema, we compare the values of $H(x)$ at all critical points and endpoints

x	10	20	55
$H(x)$	≈ 33.55	≈ 36.58	≈ 32.02

Thus, the global minimum is at $x = 55$, and the global maximum is at $x = 20$.

(For each answer blank below, write NONE in the answer blank if appropriate.)

Answer: global min(s) at $x =$ 55

Answer: global max(es) at $x =$ 20

b. [2 points] Assuming Oren consumes at least 10 milligrams and at most 55 milligrams of caffeine, what is the shortest amount of time it could take for him to finish his assignment? *Remember to include units.*

Solution: The minimum of $H(x)$ occurs at $x = 55$, where $H(55) \approx 32.02$.

Answer: ≈ 32 **minutes**