3. [11 points] For each of the problems below, circle all of the correct answers. If none of the answer choices provided are correct, circle NONE OF THESE.

a. [4 points] Let \( s(t) = \begin{cases} t^3 + 8t^2 + 6t & \text{if } t \leq c \\ 4t^2 + 2t & \text{if } t > c \end{cases} \)

For which of the following values of \( c \) is \( s(t) \) differentiable on \((−∞, ∞)\)?

i. \(-2\)

ii. \(-\frac{2}{3}\)

iii. 0

iv. \(\frac{3}{2}\)

v. 3

vi. NONE OF THESE

b. [4 points] Suppose \( f \) and \( f' \) are differentiable for all real numbers. Let \( L(x) \) be the local linearization of \( f \) at \( x = 3 \). Suppose \( f''(x) < 0 \) for all \( 2.5 < x < 3.5 \) and \( f''(x) > 0 \) for all \( 2.5 < x < 3.5 \). Which of the following must be true?

i. \( L(3) > f(3) \)

ii. \( L(3) = f(3) \)

iii. \( L(3) < f(3) \)

iv. \( L(3.1) > f(3.1) \)

v. \( L(3.1) = f(3.1) \)

vi. \( L(3.1) < f(3.1) \)

vii. \( L(3.9) > f(3.9) \)

viii. \( L(3.9) = f(3.9) \)

ix. \( L(3.9) < f(3.9) \)

x. NONE OF THESE


c. [3 points] Suppose that \( f \) is a differentiable function on \((−∞, ∞)\) with no critical points, that both \( f \) and \( f' \) are invertible, and that \( f(4) = 7 \). Which of the following statements must be true?

i. \( f \) is an increasing function.

ii. \( f \) is a decreasing function.

iii. \( f'(4) = \frac{1}{f^{-1}(7)} \).

iv. \( f'(4) = \frac{1}{(f^{-1})'(7)} \).

v. \( (f')^{-1}(4) = \frac{1}{(f^{-1})'(7)} \).

vi. \( (f')^{-1}(7) = \frac{1}{(f^{-1})'(4)} \).

vii. \( f'(4)(f^{-1})'(4) = 1 \).

viii. \( (f'(7))^{-1} = (f^{-1})'(7) \).

ix. NONE OF THESE