

3. [11 points] For each of the problems below, circle all of the correct answers. If none of the answer choices provided are correct, circle NONE OF THESE.

a. [4 points] Let  $s(t) = \begin{cases} t^3 + 8t^2 + 6t & \text{if } t \leq c \\ 4t^2 + 2t & \text{if } t > c \end{cases}$

For which of the following values of  $c$  is  $s(t)$  differentiable on  $(-\infty, \infty)$ ?

- i.  $-2$
  - ii.  $-\frac{2}{3}$
  - iii.  $0$
  - iv.  $\frac{3}{2}$
  - v.  $3$
  - vi. NONE OF THESE
- b. [4 points] Suppose  $f$  and  $f'$  are differentiable for all real numbers. Let  $L(x)$  be the local linearization of  $f$  at  $x = 3$ . Suppose  $f'(x) < 0$  for all  $2.5 < x < 3.5$  and  $f''(x) > 0$  for all  $2.5 < x < 3.5$ . Which of the following must be true?

- i.  $L(3) > f(3)$
- ii.  $L(3) = f(3)$
- iii.  $L(3) < f(3)$
- iv.  $L(3.1) > f(3.1)$
- v.  $L(3.1) = f(3.1)$
- vi.  $L(3.1) < f(3.1)$
- vii.  $L(3.9) > f(3.9)$
- viii.  $L(3.9) = f(3.9)$
- ix.  $L(3.9) < f(3.9)$

x. NONE OF THESE

- c. [3 points] Suppose that  $f$  is a differentiable function on  $(-\infty, \infty)$  with no critical points, that both  $f$  and  $f'$  are invertible, and that  $f(4) = 7$ . Which of the following statements must be true?

- i.  $f$  is an increasing function.
- ii.  $f$  is a decreasing function.
- iii.  $f'(4) = \frac{1}{f^{-1}(7)}$ .
- iv.  $f'(4) = \frac{1}{(f^{-1})'(7)}$ .
- v.  $(f')^{-1}(4) = \frac{1}{(f^{-1})'(7)}$ .
- vi.  $(f')^{-1}(7) = \frac{1}{(f^{-1})'(4)}$ .
- vii.  $f'(4)(f^{-1})'(4) = 1$ .
- viii.  $(f'(7))^{-1} = (f^{-1})'(7)$ .

ix. NONE OF THESE