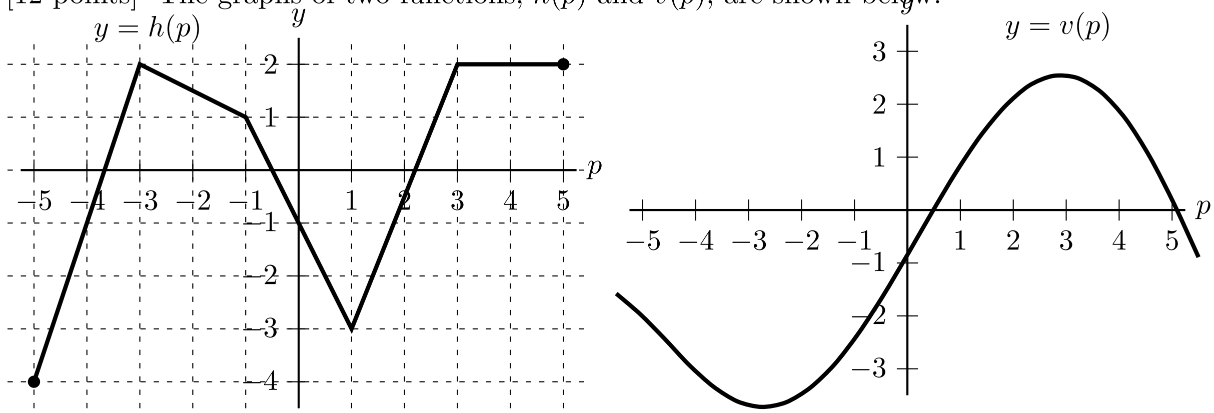
1. [12 points] The graphs of two functions, $h(p)$ and $v(p)$, are shown below.



The following questions concern the functions B, W, and Q defined as follows:

$$B(p) = \frac{h(2p)}{h(4p)}, \qquad W(p) = h(h(p)), \quad \text{and} \quad Q(p) = e^{-v(p)}.$$

Assume that the first and second derivatives of $v(p)$ are defined everywhere, i.e. that both v and v' are differentiable on $(-\infty, \infty)$. Note that the graph of $h(p)$ consists of line segments whose endpoints have integer (whole number) coordinates. Find the exact value of each of the quantities in **a.** and **b.** below. If the value does not exist, write DOES NOT EXIST. *Remember to show your work carefully.*

a. [4 points] $B'(-1)$

Solution: Applying the quotient and chain rules, we have

$$B'(p) = \frac{2h'(2p)h(4p) - 4h'(4p)h(2p)}{(h(4p))^2}.$$

So $\quad B'(-1) = \dfrac{2h'(-2)h(-4) - 4h'(-4)h(-2)}{h(-4)^2} = \dfrac{2(-\frac{1}{2})(-1) - 4(3)(\frac{3}{2})}{(-1)^2} = -17.$

Answer: $B'(-1) =$ -17

b. [4 points] $W'(2)$

Solution: By the chain rule, $W'(p) = h'(h(p))h'(p)$, so

$$W'(2) = h'(h(2))h'(2) = h'(-\frac{1}{2})h'(2) = (-2)(\frac{5}{2}) = -5.$$

Answer: $W'(2) =$ -5

c. [4 points] On the interval $-2 < p < 2$, is $Q(p)$ always increasing, always decreasing, or neither? Show your work and explain your reasoning.

Solution: By the chain rule, $Q'(p) = -v'(p)e^{-v(p)}$. Since $e^x > 0$ for all x, we know that $e^{-v(p)}$ is always positive. On the interval $-2 < p < 2$, we can see that $v(p)$ is increasing and never has a horizontal tangent line, which means that $v'(p) > 0$ on this interval. Thus $Q'(p) = -v'(p)e^{-v(p)}$ is always negative on that interval, which means that $Q(p)$ is always decreasing on this interval.