1. [12 points] The graphs of two functions, \( h(p) \) and \( v(p) \), are shown below.

The following questions concern the functions \( B, W, \) and \( Q \) defined as follows:

\[
B(p) = \frac{h(2p)}{h(4p)}, \quad W(p) = h(h(p)), \quad \text{and} \quad Q(p) = e^{-v(p)}.
\]

Assume that the first and second derivatives of \( v(p) \) are defined everywhere, i.e. that both \( v \) and \( v' \) are differentiable on \((-\infty, \infty)\). Note that the graph of \( h(p) \) consists of line segments whose endpoints have integer (whole number) coordinates. Find the exact value of each of the quantities in a. and b. below. If the value does not exist, write DOES NOT EXIST. Remember to show your work carefully.

a. [4 points] \( B'(-1) \)

\[
\begin{align*}
\text{Solution:} \quad &\text{Applying the quotient and chain rules, we have} \\
& B'(p) = \frac{2h'(2p)h(4p) - 4h'(4p)h(2p)}{(h(4p))^2}. \\
& \quad \text{So} \quad B'(-1) = \frac{2h'(-2)h(-4) - 4h'(-4)h(-2)}{h(-4)^2} = \frac{2(-\frac{1}{2})(-1) - 4(3)(\frac{3}{2})}{(-1)^2} = -17.
\end{align*}
\]

Answer: \( B'(-1) = -17 \)

b. [4 points] \( W'(2) \)

\[
\begin{align*}
\text{Solution:} \quad &\text{By the chain rule,} \quad W'(p) = h'(h(p))h'(p), \quad \text{so} \\
& W'(2) = h'(h(2))h'(2) = h'(-\frac{1}{2})h'(2) = (-2)(\frac{5}{2}) = -5.
\end{align*}
\]

Answer: \( W'(2) = -5 \)

c. [4 points] On the interval \(-2 < p < 2\), is \( Q(p) \) always increasing, always decreasing, or neither? Show your work and explain your reasoning.

\[
\begin{align*}
\text{Solution:} \quad &\text{By the chain rule,} \quad Q'(p) = -v'(p)e^{-v(p)}. \quad \text{Since} \quad e^x > 0 \quad \text{for all} \quad x, \quad \text{we know that} \\
& e^{-v(p)} \quad \text{is always positive. On the interval} \quad -2 < p < 2, \quad \text{we can see that} \quad v(p) \quad \text{is increasing} \\
& \text{and never has a horizontal tangent line, which means that} \quad v'(p) > 0 \quad \text{on this interval.} \\
& \text{Thus} \quad Q'(p) = -v'(p)e^{-v(p)} \quad \text{is always negative on that interval, which means that} \\
& \text{Q(p) is always decreasing on this interval.}
\end{align*}
\]