3. [11 points] For each of the problems below, circle all of the correct answers. If none of the answer choices provided are correct, circle NONE OF THESE.

a. [4 points] Let $s(t) = \begin{cases} t^3 + 8t^2 + 6t & \text{if } t \le c \\ 4t^2 + 2t & \text{if } t > c \end{cases}$

For which of the following values of c is $s(t)$ differentiable on $(-\infty, \infty)$?

i. $\boxed{-2}$

ii. $-\frac{2}{3}$

iii. 0

iv. $\frac{3}{2}$

v. 3

vi. NONE OF THESE

Solution: Note that the tangent lines to the graphs of $y = t^3 + 8t^2 + 6t$ and $y = 4t^2 + 2t$ are parallel at $t = c$ if and only if $3c^2 + 16c + 6 = 8c + 2$, i.e. if and only if $c = -2$ or $c = -\frac{2}{3}$. However, to be differentable at $t = c$, $s(t)$ must be continuous at $t = c$. It will be continuous when $c^3 + 8c^2 + 6c = 4c^2 + 2c$, which happens when $c = 0$ or $c = -2$. Therefore the only time the function is differentiable at c (and therefore on $(-\infty, \infty)$) is when $c = -2$.

b. [4 points] Suppose f and f' are differentiable for all real numbers. Let $L(x)$ be the local linearization of f at $x = 3$. Suppose $f'(x) < 0$ for all $2.5 < x < 3.5$ and $f''(x) > 0$ for all $2.5 < x < 3.5$. Which of the following must be true?

i. $L(3) > f(3)$

ii. $\boxed{L(3) = f(3)}$

iii. $L(3) < f(3)$

iv. $L(3.1) > f(3.1)$

v. $L(3.1) = f(3.1)$

vi. $\boxed{L(3.1) < f(3.1)}$

vii. $L(3.9) > f(3.9)$

viii. $L(3.9) = f(3.9)$

ix. $L(3.9) < f(3.9)$

x. NONE OF THESE

Solution:

c. [3 points] Suppose that f is a differentiable function on $(-\infty, \infty)$ with no critical points, that both f and f' are invertible, and that $f(4) = 7$. Which of the following statements must be true?

i. f is an increasing function.

ii. f is a decreasing function.

iii. $f'(4) = \frac{1}{f^{-1}(7)}$.

iv. $\boxed{f'(4) = \frac{1}{(f^{-1})'(7)}}$.

v. $(f')^{-1}(4) = \frac{1}{(f^{-1})'(7)}$.

vi. $(f')^{-1}(7) = \frac{1}{(f^{-1})'(4)}$.

vii. $f'(4)(f^{-1})'(4) = 1$.

viii. $(f'(7))^{-1} = (f^{-1})'(7)$.

ix. NONE OF THESE