5. [12 points] In Srebmun Foyoj, Maddy and Cal are eating lava cake. Let $T(v)$ be the time (in seconds) it takes Maddy to eat a v cm^3 serving of lava cake. Assume $T(v)$ is invertible and differentiable for $0 < v < 1000$. Several values of $T(v)$ and its first and second derivatives are given in the table below.

v	10	15	60	100	150	200	300
$T(v)$	11	22	84	194	393	513	912
$T'(v)$	2.4	1.9	1.8	3.6	3.7	0.9	17.5
$T''(v)$	-0.11	-0.08	0.05	0.04	-0.04	-0.05	0.59

Remember to show your work carefully.

a. [4 points] Use an appropriate linear approximation to estimate the amount of time it takes Maddy to eat a 64 cm^3 serving of lava cake. *Include units.*

Solution: The closest point in the table to $v = 64$ is $v = 60$, so this is the appropriate choice for the tangent line approximation. Based on the table, the line will go through $(60, 84)$ and have slope 1.8, so it must be $L(v) = 84 + 1.8(v - 60)$. Plugging in 64 for v, we get an estimate of 91.2 seconds.

Answer: 91.2 **seconds**

b. [4 points] Use the quadratic approximation of $T(v)$ at $v = 200$ to estimate $T(205)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

Solution: Let $Q(v)$ be the quadratic approximation of $T(v)$ at $v = 200$. Then

$$Q(v) = T(200)+T'(200)(v-200)+\frac{T''(200)}{2}(v-200)^2 = 513+0.9(v-200)+\frac{-0.05}{2}(v-200)^2.$$

So the resulting approximation of $T(205)$ is given by

$$T(205) \approx Q(205) = 513+0.9(205-200)-\frac{0.05}{2}(205-200)^2 = 513+4.5-0.625 = 516.875.$$

Answer: $T(205) \approx$ 516.875

c. [4 points] Let $C(v)$ be the time (in seconds) it takes Cal to eat a v cm^3 serving of lava cake, and suppose $C(v) = T(\sqrt{v})$. Let $L(v)$ be the local linearization of $C(v)$ at $v = 100$. Find a formula for $L(v)$. Your answer should not include the function names T or C.

Solution: We know $L(v) = C(100)+C'(100)(v-100)$. We also know $C(100) = T(10) = 11$. So we need to find $C'(100)$.

Since $C(v) = T(\sqrt{v})$, we apply the chain rule and see that $C'(v) = \frac{1}{2\sqrt{v}}T'(\sqrt{v})$. Using the table above, we then find that $C'(100) = \frac{1}{20}T'(10) = \frac{2.4}{20} = 0.12$.

So $L(v) = 11 + 0.12(v - 100)$.

Answer: $L(v) =$ $11 + 0.12(v - 100)$