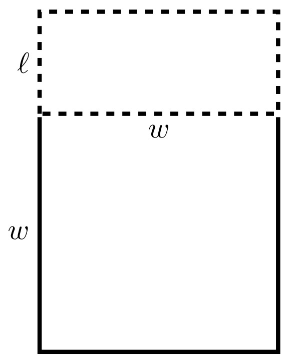
6. [11 points]
The engineer Elur Niahc has been commissioned to build a park for the citizens of Srebmun Foyoj. The park will consist of a square attached to a rectangular dog park (as shown in the diagram on the right).
The fencing for the dog park (bold, dashed line) costs \$4 per linear meter, and the fencing for the three remaining sides of the square portion of the park (bold, solid line) costs \$6 per linear meter.



a. [5 points] Assume that Elur spends \$2400 on fencing. The resulting park will have width w meters, and the length of the dog park will be ℓ meters, as shown in the diagram above. Find a formula for ℓ in terms of w.

Solution: The cost of the fencing for the dog park is $4\cdot(2\ell+2w) = 8\ell+8w$, and the cost of the fencing for the remaining three sides is $6\cdot(3w) = 18w$. So the total cost of fencing is $8\ell+8w+18w = 8\ell+26w$. Since Elur spends \$2400 on fencing, we have $2400 = 8\ell+26w$. Solving for ℓ, we find $\ell = \dfrac{2400-26w}{8} = 300 - \dfrac{13}{4}w = 300-3.25w$.

Answer: $\ell = \dfrac{2400-26w}{8} = 300-3.25w$

b. [3 points] Let $A(w)$ be the total area (in square meters) of the resulting park (including the dog park) if the width is w meters and Elur spends \$2400 on fencing. Find a formula for the function $A(w)$. The variable ℓ should not appear in your answer.
(Note: This is the function that Elur would use to find the value of w maximizing the area of the park, but you should not do the optimization in this case.)

Solution: The total area of the park in terms of w and ℓ is given by

$$A(w) = w\cdot(w+\ell) = w^2 + w\ell.$$

Using our expression for ℓ in terms of w, we find
$A(w) = w^2 + w\left(\dfrac{2400-26w}{8}\right) = w^2 + w(300-3.25w) = 300w - 2.25w^2$.

Answer: $A(w) = w^2 + w\left(\dfrac{2400-26w}{8}\right) = 300w - 2.25w^2$

c. [3 points] In the context of this problem, what is the domain of $A(w)$?

Solution: Since w is a length, $w \geq 0$. Since ℓ is also a length, $\ell = \dfrac{2400-26w}{8}$ must also be at least 0. This means the biggest w can be is when $\dfrac{2400-26w}{8} = 0$, or when

$$w = \frac{2400}{26} = \frac{1200}{13} = \frac{300}{3.25} \approx 92.3.$$

Note that if $w=0$, then the park has no area and if $\ell = 0$ then there is no dog park. In the answer shown below, we have excluded these degenerate cases.

Answer: **The interval** $\left(0, \dfrac{2400}{26}\right)$