

10. [4 points] Let a and b be constants. Consider the curve \mathcal{C} defined by the equation

$$\cos(ax) + by \ln(x) = 3 + y^3.$$

Find a formula for $\frac{dy}{dx}$ in terms of x and y . The constants a and b may appear in your answer. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Solution: We use implicit differentiation.

$$\begin{aligned}\frac{d}{dx}(\cos(ax) + by \ln(x)) &= \frac{d}{dx}(3 + y^3) \\ -a \sin(ax) + \frac{by}{x} + b \ln(x) \frac{dy}{dx} &= 3y^2 \frac{dy}{dx} \\ \frac{dy}{dx} &= \frac{\frac{by}{x} - a \sin(ax)}{3y^2 - b \ln(x)}\end{aligned}$$

Answer: $\frac{dy}{dx} =$ $\frac{\frac{by}{x} - a \sin(ax)}{3y^2 - b \ln(x)}$