

11. [6 points] Let $h(x) = x^x$. For this problem, it may be helpful to know the following formulas:

$$h'(x) = x^x (\ln(x) + 1) \quad \text{and} \quad h''(x) = x^x \left(\frac{1}{x} + (\ln(x) + 1)^2 \right).$$

a. [2 points] Write a formula for $p(x)$, the local linearization of $h(x)$ near $x = 1$.

Solution: $h(1) = 1$ and $h'(1) = 1^1(\ln(1) + 1) = 1$, so $p(x) = 1 + 1 \cdot (x - 1) = x$.

Answer: $p(x) = \underline{\hspace{10em} x \hspace{10em}}$

b. [4 points] Write a formula for $u(x)$, the quadratic approximation of $h(x)$ at $x = 1$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

Solution: $h''(1) = 1(1 + (0 + 1)^2) = 2$, so $u(x) = 1 + (x - 1) + \frac{2}{2}(x - 1)^2 = x^2 - x + 1$.

Answer: $u(x) = \underline{\hspace{10em} 1 + (x - 1) + (x - 1)^2 \hspace{10em} (= x^2 - x + 1) \hspace{10em}}$