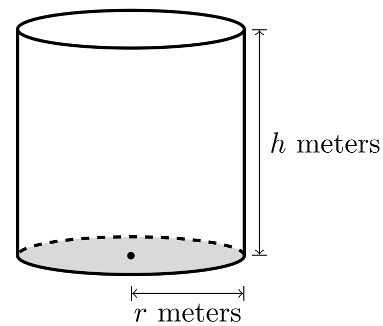
2. [10 points] Suma is making cylindrical paper cups that will be used to serve milkshakes at Qabil's Creamery. She rolls paper into a cylinder and then attaches it to the base. The thicker material that she uses for the base costs \$4.30 per square meter, and the lighter material that she uses for the vertical part of the cup costs \$2.20 per square meter. The radius of the circular base is r meters, and the height of the cup is h meters, as shown in the diagram on the right.
It may be helpful to know that the surface area of the vertical portion of the cup is $2\pi rh$.
Note: The top of the cup is left open.



Throughout this problem, assume that the material that Suma uses to make one paper cup costs \$0.12.

a. [4 points] Find a formula for h in terms of r.

Solution: The area of the vertical portion of the cup is $2\pi rh$ square meters, so the cost for the material for the vertical portion of one cup is $(2.20)(2\pi rh)$ dollars. Since the base of the cup is circular, its area is πr^2 square meters, and the cost for the material for the base of one cup is $(4.30)(\pi r^2)$ dollars. So the material that Suma uses to make one cup costs a total of $(2.20)(2\pi rh) + (4.30)(\pi r^2)$ dollars.
Therefore, we have $(2.20)(2\pi rh) + (4.30)(\pi r^2) = 0.12$.
Solving for h we find that $h = \dfrac{0.12 - 4.3\pi r^2}{4.4\pi r}$.

Answer: $h = \dfrac{0.12 - 4.3\pi r^2}{4.4\pi r}$

b. [2 points] Let $V(r)$ be the volume (in cubic meters) of the cup that Suma makes given that the material for the cup costs \$0.12 and the radius of the cup is r meters. Find a formula for $V(r)$. The variable h should not appear in your answer.
(Note: This is the function that Suma would use to find the value of r maximizing the volume of the cup, but you should not do the optimization in this case.)

Solution: Since Suma's cup is a cylinder, its volume is $\pi r^2 h$. So using what we found in part **a.** above, we see that $V(r) = \pi r^2 \cdot \dfrac{0.12 - 4.3\pi r^2}{4.4\pi r}$ which simplifies to $V(r) = \dfrac{r(0.12 - 4.3\pi r^2)}{4.4}$.

Answer: $V(r) = (\pi r^2) \cdot \dfrac{0.12 - 4.3\pi r^2}{4.4\pi r}$ **or** $\dfrac{r(0.12 - 4.3\pi r^2)}{4.4}$

c. [4 points] In the context of this problem, what is the domain of $V(r)$?

Solution: Since r is a length, it cannot be negative. Note also that if $r = 0$, then the cost of the materials for the cup would be 0 dollars (rather than \$0.12), so r must be positive. The height h also cannot be negative, and as h decreases, r increases. Therefore, r can certainly not be greater than when $h = 0$, in which case $4.3(\pi r^2) = 0.12$, so $r = \sqrt{\dfrac{0.12}{4.3\pi}}$ (since r must be positive). In this case, the cup would have height 0 and thus hold no milkshake, so we may choose to exclude this endpoint of the domain.

Answer: **the interval** $\left(0, \sqrt{\dfrac{0.12}{4.3\pi}}\right)$