3. [7 points] Consider the curve $\mathcal{D}$ defined by the equation

$$x^2y(1-y) = 9.$$

Note that the curve $\mathcal{D}$ satisfies $\dfrac{dy}{dx} = \dfrac{2xy(y-1)}{x^2(1-2y)}$.

a. [4 points] Exactly one of the following points (x, y) lies on the curve $\mathcal{D}$. Circle that one point.

$(0.9, 10)$ $\quad$ $(1, -8)$ $\quad$ $(3, 9)$ $\quad$ $(9, 3)$ $\quad$ $\boxed{(10, 0.9)}$

Then find an equation for the tangent line to the curve $\mathcal{D}$ at the point you chose.

Solution: At the point $(0.9, 10)$, the slope of the tangent line is

$$\frac{2 \cdot 10 \cdot 0.9 \cdot (0.9-1)}{100 \cdot (1 - 2 \cdot 0.9)} = \frac{1.8}{80} = \frac{9}{400} = 0.0225.$$

Answer: $y =$ $0.9 + \dfrac{1.8}{80}(x - 10) \quad (= 0.625 + 0.0225x)$

b. [3 points] Find all points on the curve $\mathcal{D}$ where the slope of the curve is undefined. Give your answers as ordered pairs. Write NONE if there are no such points.

Solution: The slope is undefined for points on $\mathcal{D}$ when the denominator of $\dfrac{dy}{dx}$ is 0. This happens when $x^2(1-2y) = 0$, so $x = 0$ or $y = \frac{1}{2}$.

When $x = 0$, we know that $x^2y(1-y) = 0$ (rather than 9), so there are no such points on the curve $\mathcal{D}$.

When $y = \frac{1}{2}$, the equation for the curve gives $x^2 \cdot \frac{1}{2}(1 - \frac{1}{2}) = 9$. So $x^2 = 36$ and therefore $x = \pm 6$. This results in the two points $(6, \frac{1}{2})$ and $(-6, \frac{1}{2})$.

Answer: $(x, y) =$ $(6, \frac{1}{2}), (-6, \frac{1}{2})$