**5**. [10 points] As a software engineer, Tendai spends many hours every day writing code. Let $w(t)$ be a function that models the number of lines of code that Tendai writes in a day if he works $t$ hours that day. Tendai works at least one hour and at most 18 hours each day. A formula for $w(t)$ is given by $w(t) = \begin{cases} -2t^2 + 28t & \text{if } 1 \le t \le 3 \\ -0.5t^2 + 9t + 43.5 & \text{if } 3 < t \le 18. \end{cases}$

**a**. [8 points] Find the values of $t$ that minimize and maximize $w(t)$ on the interval $[1, 18]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE if appropriate.

*Solution:* Note that $w$ is continuous at $t = 3$, since $\lim_{t\to 3^-} w(t) = \lim_{t\to 3^+} w(t) = 66$, so we may use the Extreme Value Theorem.
We find

$$w'(t) = \begin{cases} -4t + 28 & \text{if } 1 < t < 3 \\ -t + 9 & \text{if } 3 < t < 18. \end{cases}$$

The first expression is 0 when $t = 7$, but since this isn't in the domain of that piece, it is not a critical point. The second expression is 0 when $t = 9$.
Since both of these are polynomials, we don't have to worry about the derivative not existing on these open intervals. However, since $-4 \cdot 3 + 28 = 16$ and $-3 + 9 = 6$ are not equal, $w'$ is not defined at 3, so $t = 3$ is also a critical point.
Computing $w(t)$ at each critical point and the endpoints gives:

| $t$ | 1 | 3 | 9 | 18 |
|---|---|---|---|---|
| $w(t)$ | 26 | 66 | 84 | 43.5 |

By the Extreme Value Theorem, we therefore find that $w(t)$ attains its maximum value at $t = 9$ and its minimum at $t = 1$.

**Answer:** global max(es) at $t =$ 9

**Answer:** global min(s) at $t =$ 1

**b**. [2 points] What is the largest number of lines of code that Tendai can expect to write in a day according to this model?

*Solution:* From part **a.** we see that the maximum value of $w$ is $w(9) = 84$. So according to this model, the largest number of lines of codes that Tendai can expect to write in a day is 84.

**Answer:** 84