8. [11 points] Pepukai is studying the effect of the availability of water on the fruit productivity of Michigan apple trees. She observes that Michigan apple trees produce very few apples if they have too little water. She determines a function $p(w)$ that models the total weight, in pounds, of all the apples that an average Michigan apple tree produces in a season when it is watered with w gallons of water every week. The domain of p is $[5, 40]$. Some values of the function p and its derivative p' are shown in the table below.

w	10	15	20	25	30
$p(w)$	25	96	118	129	135
$p'(w)$	96	13	4	2	1

The function p is invertible and the functions p, p', and p^{-1} are all differentiable. Furthermore, the function p' is always decreasing.

a. [3 points] Find $(p^{-1})'(96)$.

Solution: $(p^{-1})'(96) = \frac{1}{p'(p^{-1}(96))} = \frac{1}{p'(15)} = \frac{1}{13} \approx 0.07692$

Answer: $(p^{-1})'(96) =$ $\frac{1}{13}$

b. [2 points] Circle the one statement that is best supported by the equation

$$(p^{-1})'(10) = 0.01.$$

A. To increase the total weight of apples produced in a season by an average Michigan apple tree from 10 pounds to 11 pounds, the tree should be watered with about 0.01 additional gallons of water every week.

B. If an average Michigan apple tree produces 10 pounds of apples in a season, watering the tree with 1 extra gallon every week increases the total weight of apples produced by the tree in a season by about 0.01 pounds.

C. If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 10 pounds.

D. If the amount of water that an average Michigan apple tree is watered with increases from 10 gallons every week to 10.1 gallons every week, the total weight of apples produced by the tree in a season increases by about 0.001 pounds.

c. [3 points] Write a formula for $g(w)$, the tangent line approximation to $p(w)$ near $w = 15$.

Answer: $g(w) =$ $13(w - 15) + 96$

d. [3 points] Does the tangent line approximation $g(w)$ give an underestimate or overestimate of the value of $p(w)$ at $w = 18$? Justify your answer.

Circle one: underestimate **overestimate** CANNOT BE DETERMINED

Justification:

Solution: Since p' is always decreasing, the function p is always concave down. So the tangent line at $w = 15$ lies above the graph of p and therefore the tangent line approximation gives an overestimate of the value of $p(w)$ at $w = 18$.