1. [14 points] Let g be a twice differentiable function defined on $-1 < x < 11$. Some values of $g(x)$, $g'(x)$ and $g''(x)$ are shown in the table below.

x	0	2	4	6	8	10
$g(x)$	-2	-1	3	4	5	6
$g'(x)$	0.5	2	?	5	1	2
$g''(x)$	2	1	5	-3	-1	0.5

a. [7 points] Find the *exact* value of the following expressions. If there is not enough information to compute the value, write "NI". Show all your work.

i) Let $h(x) = 2^{g(x)}$. Find $h'(6)$.

Solution: We have that $h'(x) = (\ln 2)2^{g(x)}g'(x)$ and so

$$h'(6) = (\ln 2)2^{g(6)}g'(6) = 80\ln 2$$

ii) Let $k(x) = g(x)g'(x)$. Find the value of $g'(4)$ if $k'(4) = 15$.

Solution: The product rule gives $k'(x) = g(x)g''(x) + (g'(x))^2$ and so $k'(4) = g(4)g''(4) + (g'(4))^2$. Plugging in values, $15 = 3 \cdot 5 + g'(4)^2$ and so $g'(4) = 0$.

iii) Let $r(x) = \dfrac{\sin(x)}{g(x)}$. Find $r'(0)$.

Solution: By the quotient rule, $r'(x) = \dfrac{g(x)\cos(x) - \sin(x)g'(x)}{g(x)^2}$, and so

$$r'(0) = \frac{g(0)}{g(0)^2} = -1/2.$$

b. [7 points] Let $j(x) = g(14 - 4x)$.

i) Use the values from the table to find a formula for $L(x)$, the linear approximation to $j(x)$ at $x = 2$.

Solution: We have that $j(2) = g(6) = 4$ and $j'(2) = -4g'(6) = -20$. Therefore,

$$L(x) = 4 - 20(x - 2) = -20x + 44.$$

ii) Find an approximate value for $j(2.25)$ using your formula for $L(x)$.

Solution: Using the formula from above,

$$j(2.25) \approx L(2.25) = 4 - 20(0.25) = -1.$$

iii) Is your value an overestimate or underestimate of the exact value of $j(2.25)$? Circle your answer.

OVERESTIMATE   UNDERESTIMATE   NOT ENOUGH INFORMATION