2. [12 points] Let $g(x)$ be a continuous function whose first and second derivatives are given below.

$$g'(x) = e^{2x}(2x-1)^3(x-3)^4 \qquad \text{and} \qquad g''(x) = 4e^{2x}\left(x^2-4\right)(2x-1)^2(x-3)^3$$

a. [6 points] Find all values of x at which $g(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Solution: The critical points of g are when $g'(x) = 0$, so at $x = 1/2$ and $x = 3$. Noticing that $e^{2x} > 0$ for all x, we see that:

- When $x < 1/2$, $g'(x) = (+)(-)(+) = (-)$, so g is decreasing.
- When $1/2 < x < 3$, $g'(x) = (+)(+)(+) = (+)$, so g is increasing.
- When $3 < x$, $g'(x) = (+)(+)(+) = (+)$, so g is increasing.

Therefore, g has a local minimum at $x = 1/2$, and no local maximum.

b. [6 points] Find all values of x at which $g(x)$ has an inflection point. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points. Write NONE if $g(x)$ has no points of inflection.

Solution: We see that $g''(x) = 0$ when $x = -2, 2, 1/2, 3$. We still need to check whether g changes concavity at each of these points.

- When $x < -2$, $g''(x) = (+)(+)(+)(-) = (-)$, so g is concave down.
- When $-2 < x < 1/2$, $g''(x) = (+)(-)(+)(-) = (+)$, so g is concave up.
- When $1/2 < x < 2$, $g''(x) = (+)(-)(+)(-) = (+)$, so g is concave up.
- When $2 < x < 3$, $g''(x) = (+)(+)(+)(-) = (-)$, so g is concave down.
- When $3 < x$, $g''(x) = (+)(+)(+)(+) = (+)$, so g is concave up.

Therefore g has inflection points at $x = -2, 2, 3$.