3. [10 points] Jane is designing a water tank using a cone of height \( h \) meters and a circular base of radius \( r \) meters as shown below.

\[
\begin{align*}
  r &= \text{radius} \\
  h &= \text{height} \\
  s &= \text{length of slant side}
\end{align*}
\]

a. [4 points] The cost of the material for the tank is 3 dollars per square meter for the circular base and 5 dollars per square meter for the cone (without the base). The area, \( A \), of the material used for the cone (without the base) is given by the formula \( A = \pi rs \) where \( s \) is the length of the slant side of the cone, in meters. Find a formula for \( s \) in terms of the radius \( r \) if Jane plans to spend 200 dollars on the water tank. Your answer should not include the variable \( h \).

**Solution:** The cost of the total tank is equal to the cost of the base + the cost of the cone without the base. So if Jane plans to spend $200,

\[
200 = 3(\pi r^2) + 5(\pi rs)
\]

and therefore

\[
s = \frac{200 - 3\pi r^2}{5\pi r}.
\]

b. [2 points] In the context of this problem, what are appropriate constraints on \( r \) and/or \( s \)? Choose the **one** best answer.

\[
0 < r < \infty \quad 0 < r < s \quad 0 < r < \sqrt{\frac{200}{3\pi}} \quad 0 < s < r \quad 0 < r < \sqrt{\frac{200}{5\pi}}
\]

c. [4 points] Find a formula for \( V(r) \), the volume of the tank (in cubic meters) in terms of the radius \( r \). Recall that the volume of a cone with radius \( R \) and height \( H \) is \( \frac{1}{3}\pi R^2H \). Your answer should not include the variables \( h \) and/or \( s \).

**Solution:** The volume of the tank is \( V = \frac{1}{3}\pi r^2h \). By the Pythagorean Theorem, \( s^2 = r^2 + h^2 \), and so \( h = \sqrt{s^2 - r^2} \). We also know \( s \) in terms of \( r \) from the first part of the problem, and so

\[
h = \sqrt{\left(\frac{200 - 3\pi r^2}{5\pi r}\right)^2 - r^2}.
\]

Therefore,

\[
V(r) = \frac{1}{3}\pi r^2 \sqrt{\left(\frac{200 - 3\pi r^2}{5\pi r}\right)^2 - r^2}.
\]