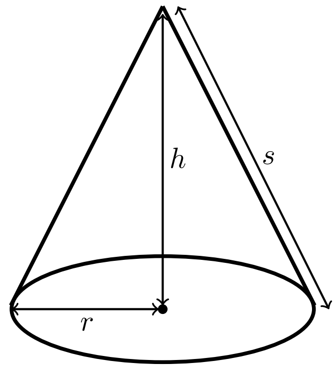
3. [10 points] Jane is designing a water tank using a cone of height h meters and a circular base of radius r meters as shown below.



r = radius
h = height
s = length of slant side

a. [4 points] The cost of the material for the tank is 3 dollars per square meter for the circular base and 5 dollars per square meter for the cone (without the base). The area, A, of the material used for the cone (without the base) is given by the formula $A = \pi r s$ where s is the length of the slant side of the cone, in meters. Find a formula for s in terms of the radius r if Jane plans to spend 200 dollars on the water tank. *Your answer should not include the variable h.*

Solution: The cost of the total tank is equal to the cost of the base + the cost of the cone without the base. So if Jane plans to spend \$200,

$$200 = 3(\pi r^2) + 5(\pi r s)$$

and therefore

$$s = \frac{200 - 3\pi r^2}{5\pi r}.$$

b. [2 points] In the context of this problem, what are appropriate constraints on r and/or s? Choose the one best answer.

$0 < r < \infty$ $\boxed{0 < r < s}$ $0 < r < \sqrt{\frac{200}{3\pi}}$ $0 < s < r$ $0 < r < \sqrt{\frac{200}{5\pi}}$

c. [4 points] Find a formula for $V(r)$, the volume of the tank (in cubic meters) in terms of the radius r. Recall that the volume of a cone with radius R and height H is $\frac{1}{3}\pi R^2 H$. *Your answer should not include the variables h and/or s.*

Solution: The volume of the tank is $V = \frac{1}{3}\pi r^2 h$. By the Pythagorean Theorem, $s^2 = r^2 + h^2$, and so $h = \sqrt{s^2 - r^2}$. We also know s in terms of r from the first part of the problem, and so

$$h = \sqrt{\left(\frac{200 - 3\pi r^2}{5\pi r}\right)^2 - r^2}.$$

Therefore,

$$V(r) = \frac{1}{3}\pi r^2 \sqrt{\left(\frac{200 - 3\pi r^2}{5\pi r}\right)^2 - r^2}.$$