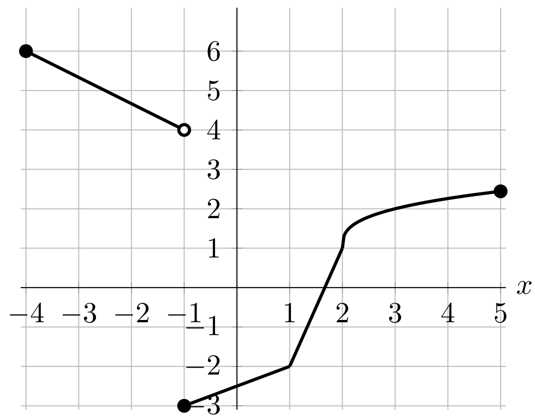
4. [15 points] Consider the graph of $h(x)$ below. Note that h is linear on the intervals $[-4,-1)$, $[-1,1]$, and $[1,2]$, differentiable on $(2,5)$, and has a sharp corner at $x=2$.

$y = h(x)$



a. [6 points] Find the exact value of the following expressions. If there is not enough information provided to find the value, write "NI". If the value does not exist, write "DNE". Show all your work.

i) Let $g(x) = xh(x)$. Find $g'(-2)$.

Solution: We have that $g'(x) = h(x) + xh'(x)$. The equation of $h(x)$ on the interval $[-4,1)$ is $h(x) = -2/3x + 10/3$, and so

$$g'(-2) = h(-2) + (-2)h'(-2) = \frac{18}{3} = 6.$$

ii) Let $p(x) = h^{-1}(x)$. Find $p'(0)$.

Solution: We have that $p'(x) = 1/(h'(h^{-1}(x))$. From the graph we see that $h^{-1}(0)$ occurs in the interval $[1,2]$, where $h(x)$ is given by the equation $h(x) = 3x-5$. Therefore, $h^{-1}(0) = 5/3$, so $p'(0) = 1/(h'(5/3))$. Since $1 < 5/3 < 2$, we see that $h'(5/3) = 3$, and so

$$p'(0) = 1/3.$$

b. [2 points] On which of the following intervals does the function $h(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle all that apply.

Solution: $[-4,-1]$ $\quad$ $[-2,1]$ $\quad$ $[0,4]$ $\quad$ $\boxed{[2,5]}$ $\quad$ NONE OF THESE

c. [3 points] On which of the following intervals does the function $h(x)$ satisfy the conclusion of the Mean Value Theorem? Circle all that apply.

Solution: $[-4,-1]$ $\quad$ $[-2,1]$ $\quad$ $\boxed{[0,4]}$ $\quad$ $\boxed{[2,5]}$ $\quad$ NONE OF THESE

d. [4 points] For which values given below is the function $m(x) = h(h(x))$ <u>not</u> differentiable? Circle all that apply.

Solution:
$\boxed{x=-3}$ $\quad$ $\boxed{x=-1}$ $\quad$ $\boxed{x=2}$ $\quad$ $\boxed{x=3}$ $\quad$ $x=4$ $\quad$ NONE OF THESE