6. [6 points] Let $L(x)$ be the linear approximation and $Q(x)$ be the quadratic approximation to the function $d(x)$ near $x = 1$. Suppose that $d'(x)$, $d''(x)$ and $d'''(x)$ are defined for all real numbers. Let $Q(x) = 7(x-1)^2 - 8(x-1) + 3$. Find the *exact* value of the following quantities. If there is not enough information to answer the question, write "NI".

Solution:

$d(0) = \text{NI}$ $\quad d'(1) = -8$ $\quad d''(1) = 14$

$L'(2) = -8$ $\quad Q'''(1) = 0$ $\quad d'''(1) = \text{NI}$

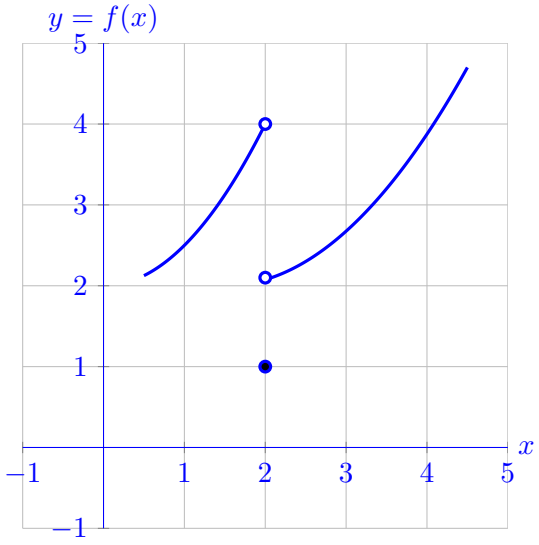
7. [5 points] Sketch graphs of functions $f(x)$ and $g(x)$ satisfying the conditions below, or circle NO SUCH FUNCTION EXISTS. You do not need to explain your answer.

Solution:

A function $f(x)$ defined on the interval $(0, 4)$ that satisfies:

i) $f'(x) > 0$ for all $x \neq 2$.

ii) $x = 2$ is a global minimum.



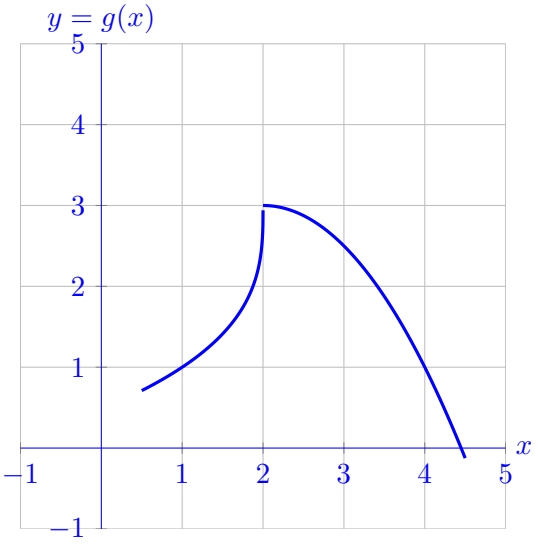
or

NO SUCH FUNCTION EXISTS

A continuous function $g(x)$ defined on the interval $(0, 4)$ that satisfies:

i) $\lim_{x \to 2^-} g'(x) = \infty$.

ii) $\lim_{x \to 2^+} g'(x) = 0$.



or

NO SUCH FUNCTION EXISTS