8. [11 points] Let \( C \) be the curve given by the equation \( 81 - (x^2 + y^2)^2 = 2xy^2 \). The graph of \( C \) is shown below.

a. [2 points] Find the coordinates \((x, y)\) of the point \( A \).

Solution: Since the point \( A \) lies at the intersection of the y-axis and the curve \( C \), then \( x = 0 \) and \( y \) satisfies \( 81 - (0^2 + y^2)^2 = 2(0)xy^2 \). Hence \( y^4 = 81 \) or \( y = 3 \).

\( A = (0, 3) \)

b. [6 points] Find \( \frac{dy}{dx} \). Show all your computations step by step.

Solution:

\[
\frac{d}{dx} (81 - (x^2 + y^2)^2) = \frac{d}{dx} (2xy^2)
\]

\[
-2(x^2 + y^2) \left( 2x + 2y \frac{dy}{dx} \right) = 2y^2 + 4xy \frac{dy}{dx}
\]

\[
-4x(x^2 + y^2) - 4y(x^2 + y^2) \frac{dy}{dx} = 2y^2 + 4xy \frac{dy}{dx}
\]

\[
-4y(x^2 + y^2) \frac{dy}{dx} - 4xy \frac{dy}{dx} = 2y^2 + 4x(x^2 + y^2)
\]

\[
\frac{dy}{dx} = -\frac{2y^2 + 4x(x^2 + y^2)}{4y(x^2 + y^2) + 4xy}
\]

c. [3 points] Find an equation of the tangent line \( L(x) \) to the graph of \( C \) at \( A \). Show all your work.

Solution: The slope of \( L(x) \) is

\[
m = -\frac{2(3)^2 + 4(0)((0)^2 + (3)^2)}{4(3)((0)^2 + (3)^2) + 4(0)(3)} = -\frac{18}{108} = -\frac{1}{6}.
\]

Hence using the point \( A \) and the slope-intercept formula for the line \( L(x) \), we get

\( L(x) = -\frac{1}{6}x + 3. \)