9. [13 points] A curve $\mathcal{C}$ is defined implicitly by the equation

$$
2 x\left(y^{2}-4 y\right)+5 x=15
$$

Note that the curve $\mathcal{C}$ satisfies

$$
\frac{d y}{d x}=\frac{-2 y^{2}+8 y-5}{4 x y-8 x}
$$

a. [4 points] Find all points on $\mathcal{C}$ with a vertical tangent line. Give your answers as ordered pairs (coordinates). Justify your answer algebraically. Write none if no such points exist.

## Answer:

b. [4 points] There is one point on $\mathcal{C}$ with a coordinate $(k, 0)$. Find the value of $k$ and write the equation of the tangent line to $\mathcal{C}$ at the point $(k, 0)$. Your equation should not include the letter $k$.

Answer: $k=$ $\qquad$ Equation of tangent line: $\qquad$
c. [5 points] Another curve $\mathcal{D}$ is defined implicitly by the equation

$$
\cos \left(W y^{3}+V x y\right)=\frac{1}{6}
$$

where $W$ and $V$ are constants. Find a formula for $\frac{d y}{d x}$ in terms of $x, y, W$, and $V$. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Answer: $\frac{d y}{d x}=$ $\qquad$
10. [7 points] The function $g$ has the property that $g(x), g^{\prime}(x)$, and $g^{\prime \prime}(x)$ are defined for all real numbers. The quadratic approximation of $g(x)$ at $x=-2$ is

$$
Q(x)=4(x+2)^{2}+\frac{1}{2}(x+2)-5 .
$$

a. [5 points] Find the exact value of each of the following quantities. If there is not enough information to answer the question, write NI.

$$
\begin{array}{ll}
g(-2)= & g^{\prime}(-2)= \\
g(0)= \\
Q^{\prime}(0)= \\
\hline
\end{array}
$$

b. [2 points] Write a formula for $L(x)$, the tangent approximation of $g(x)$ near $x=-2$.

Answer: $L(x)=$ $\qquad$

