

3. [12 points] Assume the function $h(t)$ is invertible and $h'(t)$ is differentiable. Some of the values of the function $y = h(t)$ and its derivatives are shown in the table below

t	0	1	2	3	4
$h(t)$	-2	2	3	4	8
$h'(t)$	3.5	0.5	2.5	1.5	5
$h''(t)$	6	0.25	0.3	-0.4	0.6

Use the values in the table to compute the exact value of the following mathematical expressions. If there is not enough information provided to find the value, write NI. If the value does not exist, write DNE. **Show all your work.**

- a. [3 points] Let $a(t) = h(t^2 - 5)$. Find $a'(3)$.

Solution: Since $a'(t) = 2th'(t^2 - 5)$, then $a'(3) = 6h'(4) = 6(5) = 30$

Answer: 30

- b. [3 points] Let $b(t) = \frac{h(t)}{t^2}$. Find $b'(4)$.

Solution: Since

$$b'(t) = \frac{h'(t)t^2 - 2th(t)}{t^4} \quad \text{then} \quad b'(4) = \frac{16h'(4) - 8h(4)}{256} = \frac{16(5) - 8(8)}{256} = \frac{16}{256} = \frac{1}{16}.$$

Answer: $\frac{1}{16}$

- c. [3 points] Let $c(y) = h^{-1}(y)$. Find $c'(2)$.

Solution: Since $c'(y) = \frac{1}{h'(h^{-1}(y))}$ then $c'(2) = \frac{1}{h'(h^{-1}(2))} = \frac{1}{h'(1)} = 2$.

Answer: 2

- d. [3 points] Let $g(t) = \ln(1 + 2h'(t))$. Find $g'(0)$.

Solution: Since $g'(t) = \frac{2h''(t)}{1 + 2h'(t)}$ then $g'(0) = \frac{2h''(0)}{1 + 2h'(0)} = \frac{2(6)}{1 + 2(3.5)} = \frac{12}{8} = 1.5$

Answer: 1.5