4. [12 points] Suppose $r(x)$ is a differentiable function defined for all real numbers x. The derivative and second derivative of $r(x)$ are given by

$$r'(x) = (x+1)^3(x-2)^{4/5} \qquad \text{and} \qquad r''(x) = \frac{(x+1)^2(19x-26)}{5(x-2)^{1/5}}$$

a. [6 points] Find the x-coordinates of all critical points of $r(x)$ and all values of x at which $r(x)$ has a local extremum. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write NONE if appropriate.

Solution: The critical points occur at values of x where $r'(x) = 0$ or $r'(x)$ does not exist. $r'(x)$ exists for all values of x, so the only critical points are at points where

$$r'(x) = (x+1)^3(x-2)^{4/5} = 0.$$

That is when $x = -1$ and $x = 2$.
Now we need to know the sign of $r'(x)$ on three intervals. We will consider the sign of each factor of x in order to do this.

0	sign of $(x+1)^3$	sign of $(x-2)^{4/5}$	sign of $r'(x)$
$-\infty < x < -1$	$-$	$+$	$(-)(+) = -$
$-1 < x < 2$	$+$	$+$	$(+)(+) = +$
$2 < x < \infty$	$+$	$+$	$(+)(+) = +$

This tells us that $r(x)$ is decreasing for $x < -1$ and increasing for $x > -1$. Therefore we have a local minimum at $x = -1$ and no local maxes.

Answer: Critcal points at $x = -1, 2$

Local max(es) at $x =$NONE Local min(s) at $x = -1$

b. [6 points] Find the x-coordinates of all inflection points of $r(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: The only potential inflection points occur when $r''(x) = 0$ or $r''(x)$ does not exist. From the formula, we see that $r''(x) = 0$ when $x = -1$ and $x = \frac{26}{19}$, and $r''(x)$ does not exist at $x = 2$.
Again, we need to make a table of the sign of $r''(x)$ on intervals between these points, because inflection points only occur when $r(x)$ changes concavity, which happens when $r''(x)$ changes sign.

0	sign of $(x+1)^2$	sign of $(19x-26)$	sign of $(x-2)^{1/5}$	sign of $r''(x)$
$-\infty < x < -1$	$+$	$-$	$-$	$+$
$-1 < x < \frac{26}{19}$	$+$	$-$	$-$	$+$
$\frac{26}{19} < x < 2$	$+$	$+$	$-$	$-$
$2 < x < \infty$	$+$	$+$	$+$	$+$

Therefore, $r''(x)$ changes concavity at $x = \frac{26}{19}$ and $x = 2$.

Answer: Inflection points at $x = \frac{26}{19}, 2$