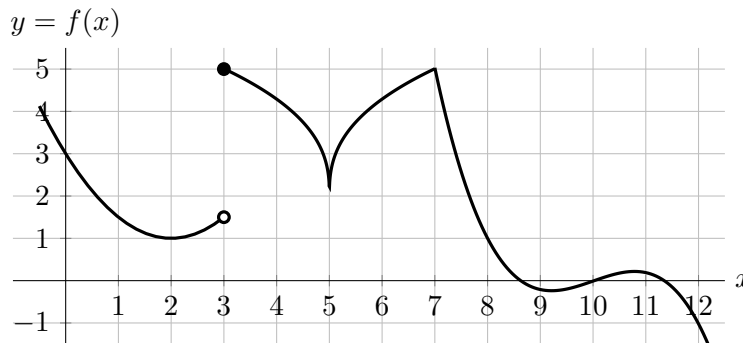**6**. [4 points] The graph of the function $f(x)$ is shown below. Note that $f(x)$ has a vertical tangent line at $x = 5$.

a. [2 points] On which of the following intervals does the function $f(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle the correct answer(s).

$\boxed{[0,2]}$ [1,3]   [2,4] $\boxed{[3,5]}$ NONE OF THESE

b. [2 points] On the interval $[8, 12]$ the hypotheses of the Mean Value Theorem are true for the function $f(x)$. What does the conclusion of this theorem say in this interval?

Answer:

Solution: There is some c on the interval $(8, 12)$ such that $f'(c) = \dfrac{f(12) - f(8)}{12 - 8} = -\dfrac{1}{2}$.

7. [5 points]

Yi is constructing a cardboard box. The base of the box will be a square of width w inches. The height of the box will be h inches. Yi will use gray cardboard for the sides of the box and brown cardboard for the bottom (the box does not have a top). Gray cardboard costs \$0.05 per square inch, while brown cardboard costs \$0.03 per square inch. Yi wants to spend \$20 on the cardboard for his box.
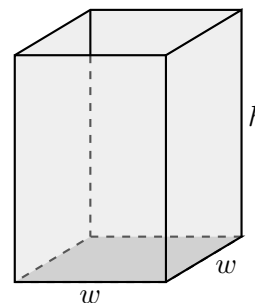
Write a formula for h in terms of w.

Solution: The area covered by the gray and the brown cardboard are $A_g = 4wh$ and $A_b = w^2$ respectively. Then the cost of the cardboard, in dollars, used in the cardboard is $C = 0.05A_g + 0,03A_b$. Hence w and h satisfy

$$C = 20 = 0.05(4wh) + 0.03w^2 = 0.2wh + 0.03w^2.$$

Then

$$h = \frac{20 - 0.03w^2}{0.2w}.$$

Answer: $h = \dfrac{20 - 0.03w^2}{0.2w}$