1. [13 points] Let g be a function such that g''(x) is defined for all real numbers. A table of values of g'(x), the derivative of g(x), is given below.

x	-5	-1	0	3	4	7
g'(x)	3	0	-4	-3	0	2

Assume that between each pair of consecutive values of x given in the table, g'(x) is either always increasing or always decreasing.

For parts **a.**–**f.**, circle <u>all</u> correct choices.

a. [1 point] At which of the following values does g(x) have a critical point?

x = -5 x = -1 x = 0 x = 3 x = 4 x = 7 None of these

b. [2 points] On which of the following intervals is g(x) always decreasing?

$$(-5, -1)$$
 $(-1, 0)$ $(0, 3)$ $(3, 4)$ $(4, 7)$ None of these

c. [2 points] At which of the following values does g(x) have a local maximum?

$$x = -1$$
 $x = 0$ $x = 3$ $x = 4$ None of these

d. [2 points] On which of the following intervals is g(x) always concave down?

$$(-5,-1)$$
 $(-1,0)$ $(0,3)$ $(3,4)$ $(4,7)$ None of these

e. [2 points] At which of the following values does g(x) have an inflection point?

$$x = -1$$
 $x = 0$ $x = 3$ $x = 4$ None of these

f. [2 points] Suppose that g(7) = 0 and g''(x) < 0 for all x > 7. Which of the following values of g(10) are possible?

$$g(10) = -5$$
 $g(10) = 2$ $g(10) = 6$ $g(10) = 11$ None of these

g. [2 points] Use the table to give the best possible estimate of g''(-3).

Solution: Since -3 is between -5 and -1, we find the average rate of g'(x) on the interval (-5, -1) to estimate the derivative of g' at -3, i.e. g''(-3):

$$\frac{3-0}{-5-(-1)} = \frac{3}{-4}$$

Answer: $g''(-3) \approx$ _____

-3

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