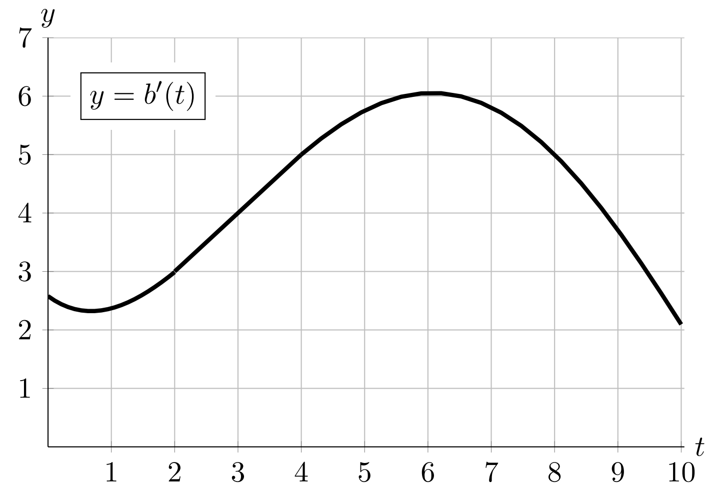
2. [9 points]

A scientist conducted an experiment in which she grew a bacterial culture in a petri dish. Let $b(t)$ be the amount of bacteria, as measured by mass in milligrams (mg), contained in the dish t hours into the experiment. A portion of the function $b'(t)$, the **derivative** of $b(t)$, is graphed to the right.

The graph of $b'(t)$ passes through the points $(2,3)$, $(3,4)$, $(4,5)$ and $(8,5)$. You may estimate any other values you need in this problem from the given graph.



a. [2 points] Using the graph, complete the following sentence.

Eight hours into the experiment, in the next ten minutes the amount of bacteria in the dish ...

(circle one) $\boxed{\text{INCREASED}}$ DECREASED by approximately 5/6 mg.

Solution: We see that $b'(4) = 5$. Since ten minutes is $1/6$ of an hour, we expect the amount of bacteria to grow by about $5 \cdot 1/6$ mg.

b. [2 points] Four hours into the experiment, there were 32.5 mg of bacteria in the dish. Write a formula for the linear approximation $L(t)$ of $b(t)$ near $t = 4$.

Solution: We know that $L(t) = b(4) + b'(4)(t-4)$. We are told that $b(4) = 32.5$ and see from the graph that $b'(4) = 5$.

Answer: $L(t) = 32.5 + 5(t-4)$

c. [2 points] Use $L(t)$ from part **b.** to estimate the amount of bacteria, in mg, in the dish at time $t = 4.3$. Is this estimate an overestimate, an underestimate, neither of these, or is there not enough information to decide?

Solution: We plug in 0.3 to the formula from **b.**. From the graph we see that $b'(t)$ is increasing near $t = 4$, so that $b''(t)$ is positive near $t = 4$. Thus $b(t)$ is concave up near $t = 4$ so this estimate is an underestimate.

Answer: Amount of bacteria at $t = 4.3$ is $\approx$ $32.5 + 5(0.3) = 34$ mg

Circle one: OVERESTIMATE $\boxed{\text{UNDERESTIMATE}}$ NEITHER NOT ENOUGH INFO

d. [3 points] Three hours into the experiment, there were 28 mg of bacteria in the dish. Write a formula for the quadratic approximation $Q(t)$ of $b(t)$ near $t = 3$.

Solution: We know that $Q(t) = b(3) + b'(4)(t-3) + \frac{b''(3)}{2}(t-3)^2$. We are told that $b(3) = 28$ and see from the graph that $b'(3) = 4$. We also find the slope of the given graph to find that $b''(3) = 1$.

Answer: $Q(t) = 28 + 4(t-3) + \frac{1}{2}(t-3)^2$