3. [12 points] Suppose $q(x)$ is a differentiable function defined for all real numbers x. The <u>derivative</u> and <u>second derivative</u> of $q(x)$ are given by

$$q'(x) = x^{2/3}(x-3)^{5/3}(x+5) \qquad \text{and} \qquad q''(x) = \frac{10(x-3)^{2/3}(x-1)(x+3)}{3x^{1/3}}.$$

a. [1 point] Find the x-coordinates of all critical points of $q(x)$. If there are none, write NONE.

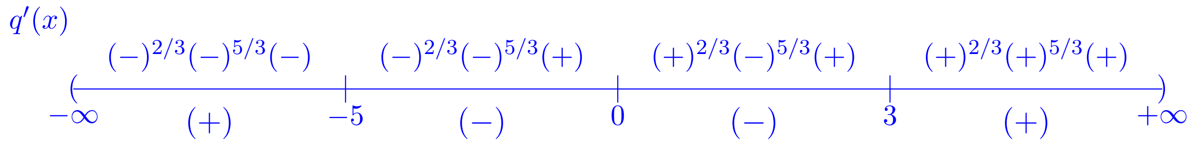
Answer: Critical point(s) of $q(x)$ at $x =$ $x = 0,\ 3,$ **and** -5

b. [2 points] Find the x-coordinates of all critical points of $q'(x)$. If there are none, write NONE.

Answer: Critical point(s) of $q'(x)$ at $x =$ $x = -3,\ 0,\ 1,$ **and** 3

c. [5 points] Find the x-coordinates of all local maxima and local minima of $q(x)$. If there are none of a particular type, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

Solution: The sign of $q'(x)$ can only possibly change at the three critical points of $q(x)$, namely 0, 3, and -5. Below is a number line showing these signs, with accompanying sign logic in order to justify how we know when $q'(x)$ is positive and when it is negative.
(Note that $a^{2/3} = (a^2)^{1/3}$, which is positive for any nonzero number a and that the sign of $a^{5/3}$ is the same as the sign of a.)



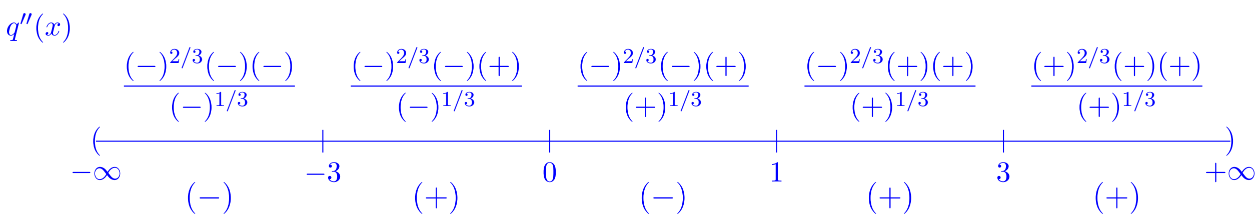
By the First Derivative Test, $q(z)$ has a local maximum at $x = -5$ and a local minimum at $x = 3$.

Note: The Second Derivative Test can be used to show that $q(x)$ has a local maximum at $x = -5$ but cannot be used to determine the bahavior of $q(x)$ at $x = 0$ or $x = 3$.

Answer: Local max(es) at $x =$ -5 and Local min(s) at $x =$ 3

d. [4 points] Find the x-coordinates of all inflection points of $q(x)$. If there are none, write NONE. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all inflection points.

Solution: The concavity of $q(x)$ can only possibly change at $x = -3,\ 0,\ 1$ and 3. Below is a number line showing the signs of $q''(x)$, which shows when the sign of $q''(x)$ actually changes, with accompanying sign logic in order to justify how we know when $q''(x)$ is positive and when it is negative.



The sign of $q''(x)$ (and hence the concavity of $q(x)$) changes at $x = -3$, 0, and 1.

Answer: Inflection point(s) at $x =$ $-3,\ 0,$ **and** 1