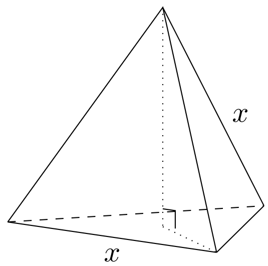
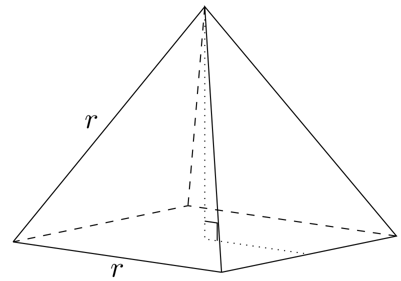
5. [7 points] An alien is building the wire frames of two pyramids. One has a base that is an equilateral triangle with side length x meters, and the other has a base that is a square with side length r meters. These shapes are shown below. For both, all triangular faces are equilateral.



Triangular Pyramid



Square Pyramid

The alien has 2 meters of wire available to build the frames, and **will use all of it**.

a. [2 points] Find a formula for r in terms of x.

Solution: There are 6 sides of length x meters, and 8 sides of length r meters. In total, these lengths must add up to 2 meters, so $6x + 8r = 2$. We can then solve for r in terms of x and find that $r = \frac{2-6x}{8}$.

Answer: $r = \dfrac{2-6x}{8}$

b. [3 points] Find a formula for $A(x)$, the combined surface area of the two pyramids (i.e. the total area of all sides and bases of both shapes). Your formula should be in terms of x only.

Recall that the area of an equilateral triangle with side length L is $\dfrac{\sqrt{3}}{4}L^2$.

Solution: On the triangular pyramid, there are 4 equilateral triangles each with side length x. On the square pyramid, there are 4 equilateral triangles each with side length r, plus one square of side length r on the base. Adding up the areas of these shapes, we find

$$\text{Total Surface Area} = 4\left(\frac{\sqrt{3}}{4}x^2\right) + 4\left(\frac{\sqrt{3}}{4}r^2\right) + r^2 = \sqrt{3}x^2 + \sqrt{3}r^2 + r^2 = \sqrt{3}x^2 + (\sqrt{3}+1)r^2.$$

We substitute $r = \dfrac{2-6x}{8}$ and find $A(x) = \sqrt{3}x^2 + \left(\sqrt{3}+1\right)\left(\frac{2-6x}{8}\right)^2$

Answer: $A(x) = \sqrt{3}x^2 + (\sqrt{3}+1)\left(\dfrac{2-6x}{8}\right)^2$

c. [2 points] The alien wants to actually build one of each type of pyramid. In the context of the problem, what is the domain of the function $A(x)$ from part **b.**? You may give your answer as an interval or using inequalities.

Solution: We must have $x > 0$ in order to get a triangular pyramid. We also need $r > 0$ to get a square pyramid, which in terms of x means

$$\frac{2-6x}{8} > 0 \quad \text{which simplifies to} \quad 2 > 6x, \quad \text{so} \quad \frac{1}{3} > x$$

Answer: $\left(0, \dfrac{1}{3}\right)$