6. [6 points] Let $K(x)$ be the concentration of krypton gas, in parts per million, at a height of x miles above the surface of a certain alien planet. Formulas for $K(x)$ and $K'(x)$ are given below.

$$K(x) = \frac{(4x-2)^4}{e^{(x+0.5)^2}} \qquad \text{and} \qquad K'(x) = \frac{-16(2x-1)^3(2x-3)(2x+3)}{e^{(x+0.5)^2}}$$

For $x \geq 0$, find the heights at which the concentration of krypton is the smallest and the largest. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers. For each answer blank, write NONE if appropriate.

Solution: The critical points of $K(x)$ are $x = -1.5$, 0.5, and 1.5. Since we are optimizing on the interval $[0, \infty)$, we will exclude -1.5 from consideration. Aside from the critical points, we also need to consider the endpoint $x = 0$ and the limit as $x \to \infty$.

$K(0)$	≈ 12.461
$K(0.5)$	0
$K(1.5)$	≈ 4.689
$\lim_{x\to\infty} K(x)$	0

The largest output value is ≈ 12.461, so $K(x)$ attains its global maximum at $x = 0$, i.e. the largest concentration of krypton is found at a height of 0 miles above the the surface.

The smallest value in the table is 0. Because there are no critical points greater than 1.5, we know that the function $K(x)$ must be decreasing on the interval $[1.5, \infty)$, starting at a value of ≈ 4.689, and decreasing towards (but never exactly achieving) a value of 0 on this interval. However, a value of 0 is actually achieved at $x = 0.5$, so $K(x)$ attains its global minimum at $x = 0.5$, i.e. the smallest concentration of krypton is found at a height of 0.5 miles above the the surface.

Answer: Smallest concentration at $x =$ 0.5 **miles**

Largest concentration at $x =$ 0 **miles**