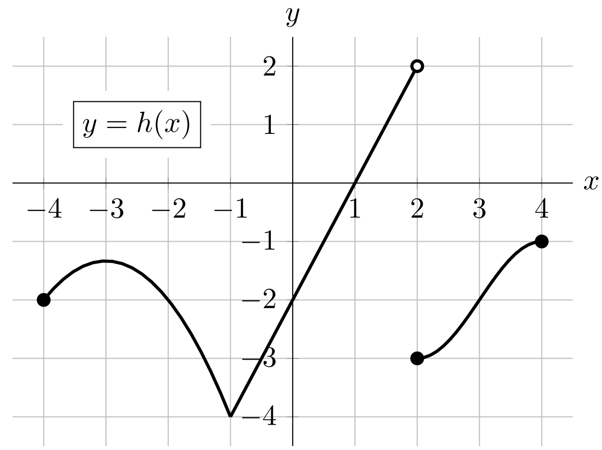
7. [11 points]
Shown to the right is the graph of a function $h(x)$.



For parts **a.–c.**, circle **all** correct choices.

a. [2 points] Which of the following are critical points of $h(x)$?

$\boxed{x = -3}$ $\boxed{x = -1}$ $x = 1$ $\boxed{x = 2}$ $x = 3$ NONE OF THESE

b. [2 points] On which of the following interval(s) does $h(x)$ satisfy the hypotheses of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $[-4, 0]$ $[0, 2]$ $\boxed{[3, 4]}$ NONE OF THESE

c. [2 points] On which of the following interval(s) does $h(x)$ satisfy the conclusion of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $\boxed{[-4, 0]}$ $[0, 2]$ $\boxed{[3, 4]}$ NONE OF THESE

d. [5 points] Define the function $k(x)$ such that

$$k(x) = \begin{cases} h(x) & -4 \le x < 1 \\ A^2 \sin(Ax + B) & 1 \le x \le 4, \end{cases}$$

where A and B are constants. Find one pair of values for A and B that make $k(x)$ differentiable at $x = 1$. *Show your work.*

Solution: For k to be differentiable, it must be continuous. At $x = 1$, continuity implies that

$$0 = A^2 \sin(A + B), \quad \text{so} \quad A = 0 \text{ or } \sin(A + B) = 0.$$

We also need the slope of each piece to match at $x = 1$, that is,

$$h'(1) = A^2 \left(\cos(A + B) \cdot A\right), \quad \text{so} \quad 2 = A^3 \cos(A + B).$$

Notice that we can rule out the possibility that $A = 0$ (since $2 \neq 0$), which forces us to choose $\sin(A + B) = 0$. The problem only asks for *one* pair of values, and *one* way to get $\sin(A + B) = 0$ is to set $A + B = 0$ so $A = -B$. This means

$$2 = A^3 \cos(0) = A^3, \quad \text{which we can solve to find} \quad A = \sqrt[3]{2} \quad \text{and} \quad B = -A = -\sqrt[3]{2}.$$

Note: There are other possible values. We could have also chosen $A + B = n\pi$ where n is any integer. If n is even, this gives $(A, B) = \left(\sqrt[3]{2}, n\pi - \sqrt[3]{2}\right)$. If n is odd, this gives $(A, B) = \left(-\sqrt[3]{2}, n\pi + \sqrt[3]{2}\right)$.

Answer: $A =$ $\sqrt[3]{2}$ and $B =$ $-\sqrt[3]{2}$