8. [11 points] Parts **a.** and **b.** are unrelated.

a. [6 points] Windchill is the temperature felt on exposed skin due to the combination of air temperature and wind speed. For a certain fixed air temperature, we define the following functions W and T.

- $W(s)$ is the windchill, in degrees Fahrenheit, when the wind speed is s miles per hour (mph).
- $T(r)$ is the time, in minutes, it takes for frostbite to develop on exposed skin when the windchill is r degrees Fahrenheit.

The functions W and T are both invertible and differentiable. Suppose that

- $W(25) = -37$
- $W'(25) = -0.4$
- $T(-25) = 25$
- $T'(-25) = 2$
- $T(-37) = 10$
- $T'(-37) = 0.75$

i. [2 points] Write an equation for the linear approximation $L(s)$ of $W(s)$ near $s = 25$.

Answer: $L(s) =$ $-37 - 0.4(s - 25)$

ii. [1 points] How many minutes does it take for frostbite to develop if the wind speed is 25 mph?

Answer: 10

iii. [3 points] If the wind speed is 26 mph, estimate the amount of time, in minutes, it takes for frostbite to develop.

Solution: To estimate $T(W(26))$, one option is to use the linear approximation of $T(W(s))$ near $s = 25$. By the chain rule, $\frac{d}{ds}(T(W(s)) = T'(W(s))W'(s)$, so this linear approximation is

$$T(W(25)) + T'(W(25))W'(25)(s-25) = 10 + (0.75)(-0.4)(s-25)$$
$$= 10 - 0.3(s-25).$$

So $T(W(26)) \approx 10 - 0.3(26 - 25) = 9.7$.

Answer: 9.7

b. [5 points] Let $A(t)$ be the temperature, in degrees Fahrenheit (°F), at time t hours after midnight on a certain winter day in Ann Arbor. You are given the following information.

- $A(t)$ is differentiable and has only one critical point on $0 < t < 12$.
- The coldest temperature that day was -4°F, which occurred at 5:00 AM.
- Between midnight and 5:00 AM, the temperature fell at an average rate of 2°F per hour.
- The temperature was increasing the fastest at 8:00 AM.
- The global maximum value of $A(t)$ on $0 \le t \le 12$ is 12°F.

On the axes below, sketch a possible graph of $A(t)$ on $0 \le t \le 12$.