8. [11 points] Parts a. and b. are unrelated.

a. [6 points] Windchill is the temperature felt on exposed skin due to the combination of air temperature and wind speed. For a certain fixed air temperature, we define the following functions $W$ and $T$.

- $W(s)$ is the windchill, in degrees Fahrenheit, when the wind speed is $s$ miles per hour (mph).
- $T(r)$ is the time, in minutes, it takes for frostbite to develop on exposed skin when the windchill is $r$ degrees Fahrenheit.

The functions $W$ and $T$ are both invertible and differentiable. Suppose that

- $W(25) = -37$
- $W'(25) = -0.4$
- $T(-25) = 25$
- $T'(-25) = 2$
- $T(-37) = 10$
- $T'(-37) = 0.75$

i. [2 points] Write an equation for the linear approximation $L(s)$ of $W(s)$ near $s = 25$.

**Answer:** $L(s) = \frac{-37 - 0.4(s - 25)}{}$

ii. [1 points] How many minutes does it take for frostbite to develop if the wind speed is 25 mph?

**Answer:** 10

iii. [3 points] If the wind speed is 26 mph, estimate the amount of time, in minutes, it takes for frostbite to develop.

**Solution:** To estimate $T(W(26))$, one option is to use the linear approximation of $T(W(s))$ near $s = 25$. By the chain rule, $\frac{d}{ds}(T(W(s))) = T'(W(s))W'(s)$, so this linear approximation is

$$T(W(25)) + T'(W(25))W'(25)(s - 25) = 10 + (0.75)(-0.4)(s - 25) = 10 - 0.3(s - 25).$$

So $T(W(26)) \approx 10 - 0.3(26 - 25) = 9.7$.

**Answer:** 9.7

b. [5 points] Let $A(t)$ be the temperature, in degrees Fahrenheit ($\circ F$), at time $t$ hours after midnight on a certain winter day in Ann Arbor. You are given the following information.

- $A(t)$ is differentiable and has only one critical point on $0 < t < 12$.
- The coldest temperature that day was $-4\circ F$, which occurred at 5:00 AM.
- Between midnight and 5:00 AM, the temperature fell at an average rate of $2\circ F$ per hour.
- The temperature was increasing the fastest at 8:00 AM.
- The global maximum value of $A(t)$ on $0 \leq t \leq 12$ is $12\circ F$.

On the axes below, sketch a possible graph of $A(t)$ on $0 \leq t \leq 12$. 

![Graph of A(t)](image-url)