9. [6 points] The implicit curve $C$ is given by the equation

$$y^2 - 1 = r^2 + x^2(y - r)$$

for some constant $r$. A graph of the curve with $r = 1$ is shown to the right. Note that

$$\frac{dy}{dx} = \frac{2x(y - r)}{2y - x^2}.$$ 

Answer each of the following questions about the implicit curve $C$. Your answers must be in exact form.

a. [2 points] When $r = 1$, the curve $C$ passes through the point $(\sqrt{2}, 0)$. Write a formula for the tangent line to the curve $C$ at this point.

**Solution:** The slope at $(\sqrt{2}, 0)$ when $r = 1$ is

$$\frac{2\sqrt{2}(0 - 1)}{2(0) - (\sqrt{2})^2} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

**Answer:**

$$y = \sqrt{2}(x - \sqrt{2})$$

b. [4 points] In this part, we do not assume anything about $r$. In particular, do not assume $r = 1$. Find the $(x, y)$ coordinates of all points at which the tangent line to the curve $C$ is horizontal. If there are no such points, write NONE. Your answer may be in terms of the constant $r$. You must show every step of your work.

**Solution:** For the tangent line to $C$ at $(x, y)$ to be horizontal, we need the numerator of $dy/dx$ to equal zero:

$$2x(y - r) = 0,$$

meaning $x = 0$ or $y = r$.

If there was a point on the curve with $y = r$, then we would have the equation

$$r^2 - 1 = r^2 + x^2(r - r),$$

$$r^2 - 1 = r^2,$$

$$-1 = 0,$$

so there are no points with $y = r$. To find points with $x = 0$, we solve

$$y^2 - 1 = r^2 + (0)^2(y - r),$$

$$y^2 = r^2 + 1,$$

$$y = \pm \sqrt{r^2 + 1}.$$

**Answer:** $(0, \sqrt{r^2 + 1})$ and $(0, -\sqrt{r^2 + 1})$