4. [13 points] Suppose $p(x)$ is a continuous function defined for all real numbers x. The **derivative** and **second derivative** of $p(x)$ are given by

$$p'(x) = |x|\,(x+4)^3 \qquad \text{and} \qquad p''(x) = \frac{4x(x+1)(x+4)^2}{|x|}.$$

Throughout this problem, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [1 point] Find the x-coordinates of all critical points of $p(x)$. If there are none, write NONE.

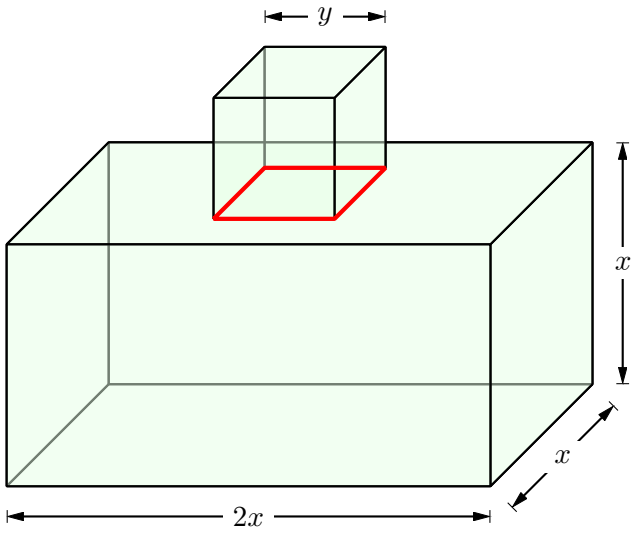
b. [2 points] Find the x-coordinates of all critical points of $p'(x)$. If there are none, write NONE.

c. [5 points] Find the x-coordinates of
i. all local minima of $p(x)$ and
ii. all local maxima of $p(x)$.
If there are none of a particular type, write NONE.

d. [5 points] Find the x-coordinates of all inflection points of $p(x)$. If there are none, write NONE.

5. [10 points] An architect is building a model out of wire and paper.

- The lower part is a box of length $2x$ centimeters (cm), depth x cm, and height x cm.
- The top part is a cube of side length y cm.
- The top part is attached to the lower part at the center of the top of the lower part.
- The architect requires that $0 \le y \le x$.
- Paper will cover the outside of the model: there is paper on the sides of the upper and lower parts, including the bottom, but no paper where the upper and lower parts meet.



The architect will use exactly 160 cm^2 of paper to make the model.

a. [4 points] Write a formula for y in terms of x.

b. [2 points] Write a formula for the function $V(x)$ which gives the total volume of the model in terms of x only.

c. [4 points] In the context of this problem, what is the domain of $V(x)$?