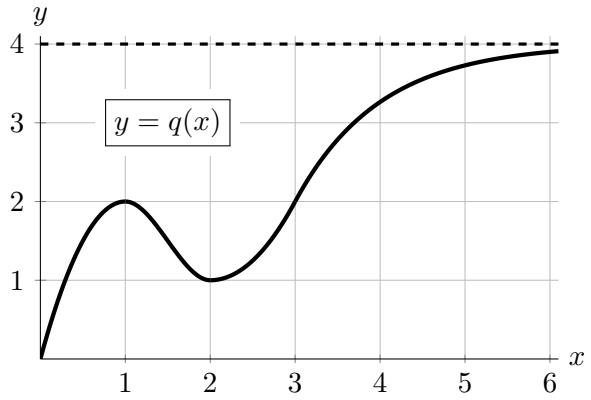
8. [10 points]

The graph of a function $q(x)$ is shown on the right. Note that:

- the domain of $q(x)$ is $[0, \infty)$,
- $q(x)$ has critical points at $x = 1$ and $x = 2$,
- $q(x)$ has no critical points for $x \geq 4$, and
- $q(x)$ has a horizontal asymptote at $y = 4$.



Now consider the piecewise-defined function $r(x)$ given as follows, where $q(x)$ is as given above:

$$r(x) = \begin{cases} \frac{1}{2}x^3 - \frac{3}{2}x & \text{if } x \leq 0 \\ q(x) & \text{if } x > 0 \end{cases}.$$

For each part below, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [5 points] Find the x-coordinates of

i. the global minimum(s) of $r(x)$ on $[-1, 3]$ and

ii. the global maximum(s) of $r(x)$ on $[-1, 3]$.

If there are none of a particular type, write NONE.

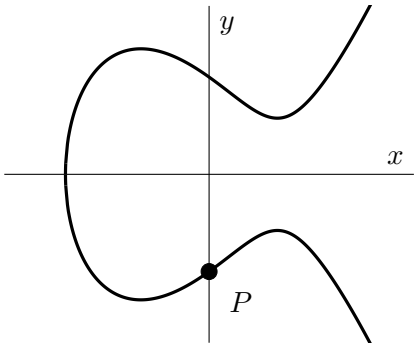
b. [5 points] Find the x-coordinates of

i. the global minimum(s) of $r(x)$ on $(-\infty, \infty)$ and

ii. the global maximum(s) of $r(x)$ on $(-\infty, \infty)$.

If there are none of a particular type, write NONE.

9. [9 points] Let $\mathcal{C}$ be the curve given by the equation $y^2 + 3x = x^3 + 3$. The graph of $\mathcal{C}$ is shown below.



Note that $\dfrac{dy}{dx} = \dfrac{3x^2 - 3}{2y}$. You must show all of your work in this problem.

a. [2 points] Find the coordinates of the point P.

b. [3 points] The point $(-2, 1)$ is on the curve $\mathcal{C}$. Find the equation of the tangent line to the curve $\mathcal{C}$ at this point.

c. [4 points] Find all points on the curve $\mathcal{C}$ where the tangent line is horizontal. Give your answer as a list of ordered pairs. Write NONE if there are no such points.