10. [9 points] A box is to be constructed with a square base. The height of the box and the side length of the square base must add up to 3 meters (m).

- Find the height and side length of the square base, in m, that lead to a box of maximum volume.
- What is the maximum volume in this case, in cubic meters?

In your solution, make sure to carefully define any variables and functions you use, use calculus to justify your answers, and show enough evidence that the values you find do in fact maximize the volume.

Solution:
Let h be the height and x the side length of the square base. We must have $h + x = 3$.

Then the total volume is

$$V(x) = x^2 h = x^2(3 - x).$$

We want to maximize $V(x)$ on the domain $0 \leq x \leq 3$. Since the derivative is

$$V'(x) = 6x - 3x^2 = 3x(2 - x),$$

the critical points are $x = 0$ and $x = 2$.

Note that the volume is zero when $x = 0$ or $x = 3$, but positive for $x = 2$. Or, we could show that $x = 2$ is a local maximum and note that, since it's the only critical point on (the interior of) our domain $[0, 3]$, it must be the global max.

Thus the maximum occurs when $x = 2$ cm and $h = 1$ cm, then the maximum volume is $2^2(3-1) = 4$.