

12. [10 points] Again suppose that $C = h(T)$ is the daily cost, in dollars, to heat a certain house if the average outside temperature that day is T degrees Fahrenheit ($^{\circ}\text{F}$). Some values of $h(T)$ and its derivative $h'(T)$ are given in the table below.

T	5	8	18	30	55
$h(T)$	8	7.2	5	3.3	1.4
$h'(T)$	-0.3	-0.25	-0.2	-0.11	-0.05

The function $h(T)$ is invertible and differentiable. Also, $h''(T)$ exists and is positive for all T .

- a. [2 points] Find the linear approximation $L(T)$ of $h(T)$ near $T = 8$.

Solution: $L(T) = 7.2 - 0.25(T - 8)$

- b. [1 point] Use your formula for $L(T)$ to approximate $h(10)$.

Solution: $h(10) \approx 7.2 - 0.25 \cdot 2 = 6.7$

- c. [2 points] Is your answer in part **b.** an overestimate or underestimate of the actual value, or is there not enough information to decide? Briefly explain.

Solution: The approximation is an underestimate since $h''(T)$ is positive, so $h(T)$ is concave up.

- d. [3 points] Suppose that the quadratic approximation $Q(T)$ of $h(T)$ near $T = 25$ is given by

$$Q(T) = 3.9 - 0.15(T - 25) + 0.003(T - 25)^2.$$

Find the values of $h(25)$, $h'(25)$, and $h''(25)$.

Solution:

$$h(25) = 3.9$$

$$h'(25) = -0.15$$

$$h''(25) = 0.003 \cdot 2 = 0.006$$

- e. [2 points] Use the table to compute $(h^{-1})'(5)$.

Solution: $(h^{-1})'(5) = \frac{1}{h'(18)} = -5$