3. [6 points] The function $g(x)$ is given by the equation

$$g(x) = \begin{cases} ax^2 & x \leq 1 \\ b - \ln(3x) & x > 1 \end{cases}$$

where a and b are constants. Find one pair of **exact** values for a and b such that $g(x)$ is differentiable, or write NONE if there are none. Be sure your work is clear.

Solution:
Continuity at $x = 1$ requires:

$$a(1^2) = b - \ln(3 \cdot 1)$$
$$a = b - \ln(3).$$

So $b = a + \ln(3)$.

Note that $\frac{d}{dx}(ax^2) = 2ax$ and $\frac{d}{dx}\left(b - \ln(3x)\right) = -\frac{1}{x}$
So differentiability at $x = 1$ also requires:

$$2a(1) = -\frac{1}{1}$$
$$2a = -1$$
$$a = -\frac{1}{2}.$$

Therefore, $a = -\frac{1}{2}$ and $b = -\frac{1}{2} + \ln 3$.