4. [13 points] Suppose $p(x)$ is a continuous function defined for all real numbers x. The **derivative** and **second derivative** of $p(x)$ are given by

$$p'(x) = |x|\,(x+4)^3 \qquad \text{and} \qquad p''(x) = \frac{4x(x+1)(x+4)^2}{|x|}.$$

Throughout this problem, you must use calculus to find and justify your answers. Make sure your final conclusions are clear, and that you show enough evidence to justify those conclusions.

a. [1 point] Find the x-coordinates of all critical points of $p(x)$. If there are none, write NONE.

Solution: $x = -4,\ 0$.

b. [2 points] Find the x-coordinates of all critical points of $p'(x)$. If there are none, write NONE.

Solution: $x = -4,\ -1,\ 0$.

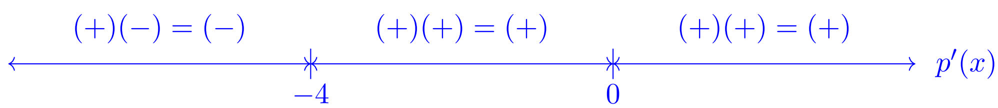
c. [5 points] Find the x-coordinates of

i. all local minima of $p(x)$ and

ii. all local maxima of $p(x)$.

If there are none of a particular type, write NONE.

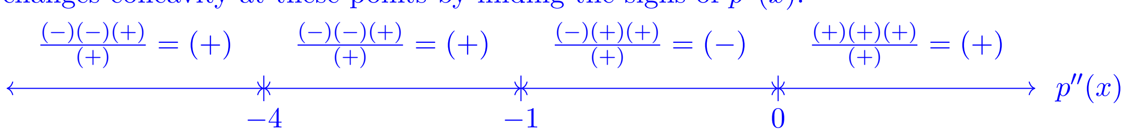
Solution: The second derivative test is inconclusive, so we have to use the first derivative test. Here's a number line showing a sign-logic calculation for the test:



(We could also compute $p'(x)$ at values of x between critical points.) We can conclude that $p(x)$ has a local minimum at $x = -4$ and no local maximum.

d. [5 points] Find the x-coordinates of all inflection points of $p(x)$. If there are none, write NONE.

Solution: We know from part b that the candidate points are $-4, -1$, and 0, so we test whether $p(x)$ changes concavity at these points by finding the signs of $p''(x)$:



So $p(x)$ has an inflection point at $x = -1$ and $x = 0$.