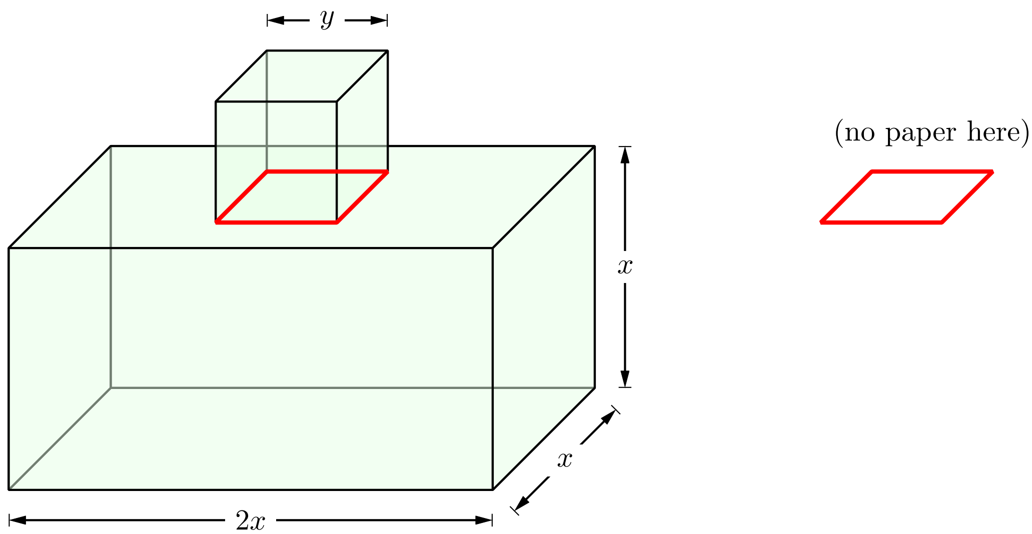
5. [10 points] An architect is building a model out of wire and paper.

- The lower part is a box of length $2x$ centimeters (cm), depth x cm, and height x cm.
- The top part is a cube of side length y cm.
- The top part is attached to the lower part at the center of the top of the lower part.
- The architect requires that $0 \le y \le x$.
- Paper will cover the outside of the model: there is paper on the sides of the upper and lower parts, including the bottom, but no paper where the upper and lower parts meet.



The architect will use exactly 160 cm^2 of paper to make the model.

a. [4 points] Write a formula for y in terms of x.

Solution: The surface area of the lower part is $4 \cdot 2x^2 + 2 \cdot x^2 = 10x^2$, but we should subtract y^2 to account for the hole in the top face. The surface area of the sides and top (but without the bottom) of the top part is $5y^2$. So the total amount of paper needed is $10x^2 - y^2 + 5y^2 = 10x^2 + 4y^2 = 160$. So $y = \sqrt{40 - \frac{10}{4}x^2}$.

b. [2 points] Write a formula for the function $V(x)$ which gives the total volume of the model in terms of x only.

Solution: The total volume is $2x^3 + y^3 = 2x^3 + \left(\sqrt{40 - \frac{10}{4}x^2}\right)^3$.

c. [4 points] In the context of this problem, what is the domain of $V(x)$?

Solution:
The value of x would be largest when $y = 0$, in which case $10x^2 + 4(0)^2 = 160$, so that $x = 4$.

The smallest x can be is y, so that $10x^2 + 4x^2 = 160$ and $x = \sqrt{160/14}$.

So the domain is $\sqrt{\frac{160}{14}} \le x \le 4$.