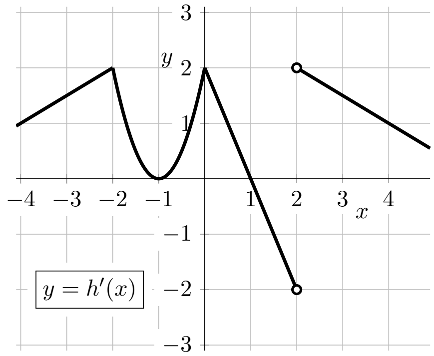
6. [14 points] A table of values for a differentiable function $g(x)$ and its derivative $g'(x)$ are shown below to the left. Below to the right is shown a portion of the graph of $h'(x)$, the <u>derivative</u> of a function $h(x)$. The function $h(x)$ is defined and continuous for all real numbers.

x	-1	0	1	3	4
$g(x)$	0	2	5	1	-7
$g'(x)$	4	3	-1	-6	-3



Answer parts **a.–b.**, or write NONE if appropriate. You do not need to show work.

a. [2 points] List the x-coordinates of all critical points of $h(x)$ on the interval $(-4, 4)$.

Solution: $x = -1, 1, 2$

b. [2 points] List the x-coordinates of all local maxima of $h(x)$ on the interval $(-4, 4)$.

Solution: $x = 1$

Find the **exact** values for parts **c.–e.**, or NEI if there is not enough information to do so. Write DNE if the value does not exist. Your answers should not include the letters g or h but you do not need to simplify. Show work.

c. [2 points] Let $A(x) = \dfrac{\sin(x) + 3}{g(x)}$. Find $A'(0)$.

Solution: $A'(x) = \dfrac{g(x)\cos(x) - (\sin(x) + 3)g'(x)}{g(x)^2}$, so $A'(0) = \dfrac{2 \cdot 1 - (0 + 3) \cdot 3}{2^2} = -\dfrac{7}{4}$.

d. [2 points] Let $f(x) = g(h'(x))$. Find $f'(4)$.

Solution: $f'(x) = h''(x)g'(h'(x))$, so $f'(4) = -\dfrac{1}{2} \cdot g'(1) = \dfrac{1}{2}$.

e. [2 points] Let $P(x) = xe^{g(x)}$. Find $P'(-1)$.

Solution: $P'(x) = e^{g(x)} + xg'(x)e^{g(x)}$, so $P'(-1) = e^0 + (-1)(4)e^0 = -3$.

Answer parts **f.–g.** You do not need to show work.

f. [2 points] Complete the following sentence.

Because the function $g(x)$ satisfies the hypotheses of the mean value theorem on the interval $[-1, 4]$, there must be some point c with $-1 \le c \le 4$ such that...

Solution: $g'(c) = -\dfrac{7}{5}$.

g. [2 points] On which of the following intervals does $h'(x)$ satisfy the hypotheses of the mean value theorem? List all correct answers, or write NONE.

$[-2, 0]$ $\quad$ $[-1, 1]$ $\quad$ $[3, 4]$

Solution: $[-2, 0]$ and $[3, 4]$