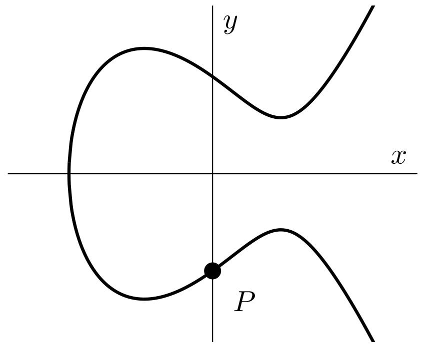
9. [9 points] Let $\mathcal{C}$ be the curve given by the equation $y^2 + 3x = x^3 + 3$. The graph of $\mathcal{C}$ is shown below.



Note that $\dfrac{dy}{dx} = \dfrac{3x^2 - 3}{2y}$. You must show all of your work in this problem.

a. [2 points] Find the coordinates of the point P.

Solution: $y^2 + 3(0) = 0^3 + 3$, so $(0, -\sqrt{3})$

b. [3 points] The point $(-2, 1)$ is on the curve $\mathcal{C}$. Find the equation of the tangent line to the curve $\mathcal{C}$ at this point.

Solution: Since the slope at this point is $\dfrac{dy}{dx} = \dfrac{3(-2)^2 - 3}{2(1)} = \dfrac{9}{2}$, the tangent line is $y = 1 + \dfrac{9}{2}(x + 2)$.

c. [4 points] Find all points on the curve $\mathcal{C}$ where the tangent line is horizontal. Give your answer as a list of ordered pairs. Write NONE if there are no such points.

Solution: To find where the tangent line is horizontal, set $3x^2 - 3 = 0$ to find $x = \pm 1$.

When $x = 1$, we find $y^2 + 3(1) = (1)^3 + 3$, or $y^2 = 1$, so $y = \pm 1$.
This leads to the points $(1, 1)$ and $(1, -1)$.

When $x = -1$, we find $y^2 + 3(-1) = (-1)^3 + 3$, or $y^2 = 5$, so $y = \pm\sqrt{5}$.
This leads to the points $(-1, \sqrt{5})$, and $(-1, -\sqrt{5})$.