2. [11 points]

a. [6 points] Let $f(x)$ be a continuous function defined for all real numbers and suppose that $f'(x)$, the derivative of $f(x)$, is given by

$$f'(x) = \frac{(x-2)(x+3)}{|x|}.$$

Find the exact x-coordinates of all local minima and local maxima of $f(x)$. If there are none of a particular type, write NONE. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: The critical points of f are where $f'(x) = 0$, which occurs at $x = 2$ and $x = -3$, and where f' DNE, which occurs at $x = 0$.

(checking signs for 1st Derivative Test)	$x < -3$	$-3 < x < 0$	$0 < x < 2$	$2 < x$
$(x-2)$	$-$	$-$	$-$	$+$
$(x+3)$	$-$	$+$	$+$	$+$
$\lvert x\rvert$	$+$	$+$	$+$	$+$
$f'(x) = \frac{(x-2)(x+3)}{\lvert x\rvert}$	$\frac{-\cdot-}{+} = +$	$\frac{-\cdot+}{+} = -$	$\frac{-\cdot+}{+} = -$	$\frac{+\cdot+}{+} = +$

This gives the following number line for $f'(x)$:

$f'(x)$: $\frac{-\cdot-}{+} = +$ | -3 | $\frac{-\cdot+}{+} = -$ | 0 | $\frac{-\cdot+}{+} = -$ | 2 | $\frac{+\cdot+}{+} = +$

By the First Derivative Test, $f(x)$ has a local max at $x = -3$ and a local min at $x = 2$. There is no local extremum at $x = 0$.

Note: Though it doesn't change the points you need to consider on your number line, there is actually no function f that satisfies these conditions! To find out why, take Math 116.

Answer: Local max(es) at $x =$ -3 and Local min(s) at $x =$ 2

b. [5 points] Let $g(x)$ be a different continuous function defined for all real numbers and suppose that $g''(x)$, the second derivative of $g(x)$, is given by

$$g''(x) = 2^x(x-1)^2(x+5).$$

Find the exact x-coordinates of all inflection points of $g(x)$, or write NONE if there are none. You must use calculus to find and justify your answers. Be sure your conclusions are clearly stated and that you show enough evidence to support them.

Solution: We start by finding any values of x for which $g''(x) = 0$ or g'' DNE, and find $x = 1$ and $x = -5$. Now we check to see whether the concavity of g changes at these points:

$g''(x)$: $+\cdot+\cdot- = -$ | -5 | $+\cdot+\cdot+ = +$ | 1 | $+\cdot+\cdot+ = +$

Answer: Inflection point(s) at $x =$ -5