

3. [15 points] Let the differentiable function $h(t)$ represent the height in inches (in) of a toy airplane above the ground at time t seconds (sec). Below is a table of some values for $h(t)$ and $h'(t)$. Assume that $h(t)$ is invertible, and that $h'(t)$ is differentiable for $t > 0$.

t	0	2	4	6	8
$h(t)$	28	19	11	8	4
$h'(t)$	-5	-4	-2	-1.5	-0.5

For parts **a.**–**d.**, you do not need to show work, but partial credit can be earned from work shown. You do not need to simplify numerical answers.

- a. [3 points] Approximate $h''(8)$. Include units.

Solution:

$$h''(8) \approx \frac{h'(8) - h'(6)}{8 - 6} = \frac{-0.5 - (-1.5)}{2} = \frac{1}{2}.$$

Answer: 0.5 in/sec²

- b. [3 points] Find a formula for the linear approximation $L(t)$ to the function $h(t)$ at $t = 2$.

Solution: $L(t) = h(2) + h'(2)(t - 2) = 19 - 4(t - 2) = -4t + 27$

Answer: $L(t) =$ $19 - 4(t - 2) = -4t + 27$

- c. [2 points] Use your answer from the previous part to approximate $h(1.9)$. Include units.

Solution: $h(1.9) \approx L(1.9) = 19 - 4(1.9 - 2) = 19 - 4(-0.1) = 19.4$

Answer: 19.4 inches

- d. [2 points] Compute the **exact** value of $(h^{-1})'(8)$. (You do not need to include units.)

Solution: $(h^{-1})'(8) = \frac{1}{h'(h^{-1}(8))} = \frac{1}{h'(6)} = \frac{1}{-1.5}.$

Answer: $\frac{1}{-1.5} = -\frac{2}{3}$

- e. [3 points] Suppose that $(h^{-1})'(3) = -9$. Complete the following sentence to give a practical interpretation of this equation.

When the toy airplane is at a height of 3 inches, to descend an additional 0.1 inches ...

Solution: ... will take about 0.9 seconds.

- f. [2 points] Note that $h(t)$ satisfies the hypotheses of the Mean Value Theorem on $[0, 8]$. Complete the following sentence about what the conclusion of this theorem implies is true.

At some time between $t = 0$ and $t = 8$, the height of the toy airplane is ...

(circle one) INCREASING DECREASING at a rate of 3 in/sec.