**5.** [8 points]  Aziz is starting a donut shop. He wants to build a display case to show off his donuts. He wants a case of width $w$ feet, height $h$ feet, and length $h$ feet, so that, as shown in the diagram below, the left and right sides are squares. The top and front of the case will be made of glass, while the square sides, back, and bottom will be made of metal. Glass costs 2 dollars per square foot, and metal costs 4 dollars per square foot. Aziz plans to spend exactly 80 dollars on the display case.
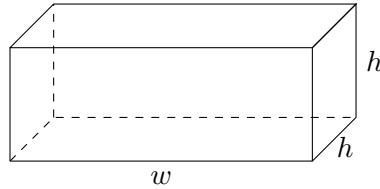


**a.** [3 points] Write a formula for $w$ in terms of $h$.

*Solution:*

$$80 = 2(2hw) + 2(4h^2) + 2(4hw)$$
$$80 - 8h^2 = w(4h + 8h)$$
$$w = \frac{80 - 8h^2}{12h} = \frac{20 - 2h^2}{3h} = \frac{20}{3h} - \frac{2h}{3}$$

**b.** [2 points] Aziz wants to maximize the volume of the display case. Find a formula for the function $V(h)$ which gives the volume in cubic feet of the display case in terms of $h$ only.
*Your formula should not include the letter $w$.*

*Solution:*

$$V = h^2 w$$
$$V(h) = h^2 \left( \frac{80 - 8h^2}{12h} \right) = \frac{80h - 8h^3}{12}$$

**c.** [3 points] What is the domain of the function $V(h)$ in the context of this problem?

*Solution:*

$$0 \le w = \frac{80 - 8h^2}{12h}$$
$$0 \le 80 - 8h^2$$
$$8h^2 \le 80$$
$$h \le \sqrt{\frac{80}{8}}$$

Note that $h$ cannot be 0. So domain is $0 < h \le \sqrt{10}$ or $(0, \sqrt{10}]$. It's also reasonable to use a domain of $0 < h < \sqrt{10}$ instead.