7. [6 points] Define the piecewise function $g(x)$ as below, where a and b are constants.

$$g(x) = \begin{cases} a + b\sin(\pi(x+2)) & x \le -2 \\ -3(x+2)+4 & x > -2 \end{cases}$$

Find one pair of **exact** values for a and b such that $g(x)$ is differentiable, or write NONE if there are none. You do not need to simplify your answers but be sure your work is clear.

Solution: First, we need $g(x)$ to be continuous at $x = -2$.

$$g(-2) = \lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} a + b\sin(\pi(x+2)) = a + b\sin(\pi(-2+2)) = a + b\sin(0) = a.$$

$$\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} -3(x+2)+4 = -3(-2+2)+4 = 4.$$

So in order for $g(x)$ to be continuous at $x = -2$, we must have $\boxed{a = 4}$ (and in that case, $g(x)$ is indeed continuous at $x = -2$).

For differentiability, we also need the slope on each side of the point at $x = -2$ to match up so that there is not a sharp corner. We have

$$g'(x) = \begin{cases} \pi \cdot b\cos(\pi(x+2)) & x < -2 \\ -3 & x > -2 \end{cases}$$

Plugging in $x = -2$ to the first piece gives $\pi \cdot b\cos(\pi(-2+2)) = \pi \cdot b\cos(0) = \pi \cdot b$. So differentiability requires $\pi \cdot b = -3$ and therefore $\boxed{b = -3/\pi}$.

Answer: $a =$ 4 and $b =$ $-3/\pi$