7. [6 points] Define the piecewise function \( g(x) \) as below, where \( a \) and \( b \) are constants.

\[
g(x) = \begin{cases} 
  a + b \sin(\pi(x + 2)) & x \leq -2 \\
  -3(x + 2) + 4 & x > -2 
\end{cases}
\]

Find one pair of **exact** values for \( a \) and \( b \) such that \( g(x) \) is differentiable, or write NONE if there are none. You do not need to simplify your answers but be sure your work is clear.

**Solution:** First, we need \( g(x) \) to be continuous at \( x = -2 \).

\[
g(-2) = \lim_{x \to -2^-} g(x) = \lim_{x \to -2^-} a + b \sin(\pi(x + 2)) = a + b \sin(\pi(-2 + 2)) = a + b \sin(0) = a.
\]

\[
\lim_{x \to -2^+} g(x) = \lim_{x \to -2^+} -3(x + 2) + 4 = -3(-2 + 2) + 4 = 4.
\]

So in order for \( g(x) \) to be continuous at \( x = -2 \), we must have \( a = 4 \) (and in that case, \( g(x) \) is indeed continuous at \( x = -2 \)).

For differentiability, we also need the slope on each side of the point at \( x = -2 \) to match up so that there is not a sharp corner. We have

\[
g'(x) = \begin{cases} 
  \pi \cdot b \cos(\pi(x + 2)) & x < -2 \\
  -3 & x > -2 
\end{cases}
\]

Plugging in \( x = -2 \) to the first piece gives \( \pi \cdot b \cos(\pi(-2 + 2)) = \pi \cdot b \cos(0) = \pi \cdot b \). So differentiability requires \( \pi \cdot b = -3 \) and therefore \( b = -3/\pi \).

**Answer:** \( a = \underline{4} \) and \( b = -3/\pi \).