8. [8 points]

a. [5 points] Consider the curve $\mathcal{C}$ defined by the equation $\ln(x^2) + y = e^{4y}$.

For this curve $\mathcal{C}$, find a formula for $\frac{dy}{dx}$ in terms of x and y. Clearly show every step of your work.

Solution:

$$\frac{d}{dx}\left(\ln(x^2) + y\right) = \frac{d}{dx}\left(e^{4y}\right)$$

$$\frac{2x}{x^2} + \frac{dy}{dx} = 4e^{4y}\frac{dy}{dx}$$

$$\frac{2}{x} = 4e^{4y}\frac{dy}{dx} - \frac{dy}{dx}$$

$$\frac{2}{x} = \frac{dy}{dx}\left(4e^{4y} - 1\right)$$

$$\frac{2}{x\left(4e^{4y} - 1\right)} = \frac{dy}{dx}$$

b. [3 points] Let $\mathcal{D}$ be a different implicitly defined curve. The curve $\mathcal{D}$ passes through the point $(2, 1)$ and satisfies

$$\frac{dy}{dx} = \frac{-2x - y}{x + 3y^2 - 1}.$$

Write an *equation* for the tangent line to the curve $\mathcal{D}$ at the point $(2, 1)$. Show your work.

Solution: We are given the coordinates of a point on the line so need only to find the slope, which is given by $\frac{dy}{dx}$. Plugging in the point $(2, 1)$ we find that the slope of the tangent line is $\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{-2(2) - 1}{2 + 3(1)^2 - 1} = \frac{-5}{4}$ so $y = 1 - \frac{5}{4}(x - 2)$. Using point-slope form we find that an equation for the tangent line at $(2, 1)$ is

$$y = 1 - \frac{5}{4}(x - 2).$$