10. [7 points] The curve $\mathcal{C}$ is given by the equation $e^{\cos(x^2-y^2)} = ex$.

a. [2 points] Which of the following points (x, y) lie on the curve $\mathcal{C}$? Circle all correct answers.

$\boxed{(1,1)}$ $(-1,1)$ $\boxed{(1,-1)}$ $(0,\sqrt{2}\,)$ NONE OF THESE

b. [5 points] Compute $\frac{dy}{dx}$. Show every step of your work and circle your final answer.

Solution: Implicitly differentiating, we get: $e^{\cos(x^2-y^2)} \cdot \big(-\sin(x^2-y^2)\big) \cdot \left(2x - 2y\frac{dy}{dx}\right) = e.$

Now we solve for $\frac{dy}{dx}$:

$$2x - 2y\frac{dy}{dx} = \frac{e}{e^{\cos(x^2-y^2)} \cdot \big(-\sin(x^2-y^2)\big)},$$

$$-2y\frac{dy}{dx} = \frac{e}{e^{\cos(x^2-y^2)} \cdot \big(-\sin(x^2-y^2)\big)} - 2x,$$

$$\frac{dy}{dx} = \frac{e}{-2ye^{\cos(x^2-y^2)} \cdot \big(-\sin(x^2-y^2)\big)} - \frac{2x}{-2y} = \frac{e}{2ye^{\cos(x^2-y^2)}\sin(x^2-y^2)} + \frac{x}{y}.$$

Or, one can distribute the left-hand side first, to obtain the equivalent answer

$$\frac{dy}{dx} = \frac{2xe^{\cos(x^2-y^2)} \cdot \sin(x^2-y^2) + e}{2ye^{\cos(x^2-y^2)} \cdot \sin(x^2-y^2)}.$$

11. [8 points]

Suppose $h(x)$ is a function such that $h(x)$ has **exactly three** critical points. Assume that both $h(x)$ and $h'(x)$ are differentiable on $(-\infty, \infty)$. A table of values is given to the right.

x	0	3	5	7
$h(x)$	2	?	4	4
$h'(x)$	-1	0	0	?

a. [2 points] Note that $h(x)$ satisfies the hypotheses of the Mean Value Theorem on $[5, 7]$. Briefly explain why the conclusion of this theorem implies that one of the three critical points of $h(x)$ must be in the interval $5 < x < 7$.

Solution: The conclusion of the Mean Value Theorem applied to $h(x)$ on $[5, 7]$ states that there exists a point c with $5 < c < 7$ such that $h'(c) = \frac{h(7)-h(5)}{7-5} = \frac{4-4}{7-5} = 0$, making c a critical point of $h(x)$ in the interval $5 < x < 7$.

In the parts below, circle all correct answers. No justification is needed.

b. [2 points] On which of the following intervals *must* $h(x)$ be increasing on the entire interval?

$(0,3)$ $\boxed{(3,5)}$ $(5,6)$ $(6,7)$ NONE OF THESE

c. [2 points] Which of the following *could* be the x-coordinate of a global minimum of $h(x)$ on $(-\infty, \infty)$?

$x = 0$ $\boxed{x = 3}$ $x = 5$ $\boxed{x = 6}$ NONE OF THESE

d. [2 points] Also suppose that $h(x)$ is concave down on $(-\infty, 0)$. Which of the following *could* be the value of $h(-2)$?

1 2 $\boxed{3}$ 4 5 NONE OF THESE