2. [9 points] Suppose $q(t)$ is a continuous function defined for all real numbers t. The **derivative** and **second derivative** of $q(t)$ are given by

$$q'(t) = te^{t/2}|t-3| \quad \text{and} \quad q''(t) = \frac{e^{t/2}(t-3)(t-2)(t+3)}{2|t-3|}.$$

Throughout this problem, you must use calculus to find and justify your answers. Make sure you show enough evidence to justify your conclusions.

a. [5 points] Find the t-coordinates of all local minimum(s) and local maximum(s) of $q(t)$. If there are none of a particular type, write NONE.

Solution: The critical points of q are where $q'(t) = 0$, which occurs at $t = 0$ and $t = 3$. There are no points at which q' DNE.

(checking signs for 1st Derivative Test)	$t < 0$	$0 < x < 3$	$3 < t$
t	$-$	$+$	$+$
$e^{t/2}$	$+$	$+$	$+$
$\|t-3\|$	$+$	$+$	$+$
$q'(t) = te^{t/2}\|t-3\|$	$- \cdot + \cdot + = -$	$+ \cdot + \cdot + = +$	$+ \cdot + \cdot + = +$

This gives the following number line for $q'(t)$:

$q'(t)$: $- \cdot + \cdot + = -$ | 0 | $+ \cdot + \cdot + = +$ | 3 | $+ \cdot + \cdot + = +$

By the First Derivative Test, $q(t)$ has a local min at $t = 0$. There is no local extremum at $t = 3$.

Answer: Local min(s) at $t =$ 0 and Local max(es) at $t =$ **NONE**

b. [4 points] Find the t-coordinates of all inflection points of $q(t)$, or write NONE if there are none.

Solution: We start by finding any values of t for which $q''(t) = 0$ or q'' DNE, and find $t = -3$, 2, and 3. Now we need to check to see whether the concavity of q changes at these points:

$q''(t)$: $\frac{- \cdot - \cdot -}{+} = -$ | -3 | $\frac{- \cdot - \cdot +}{+} = +$ | 2 | $\frac{- \cdot + \cdot +}{+} = -$ | 3 | $\frac{+ \cdot + \cdot +}{+} = +$

Because the sign of the second derivative of q changes at each of these points, they are all inflection points.

Answer: Inflection point(s) at $t =$ **-3, 2, 3**