

2. [9 points] Suppose  $q(t)$  is a continuous function defined for all real numbers  $t$ . The **derivative** and **second derivative** of  $q(t)$  are given by

$$q'(t) = te^{t/2}|t - 3| \quad \text{and} \quad q''(t) = \frac{e^{t/2}(t - 3)(t - 2)(t + 3)}{2|t - 3|}.$$

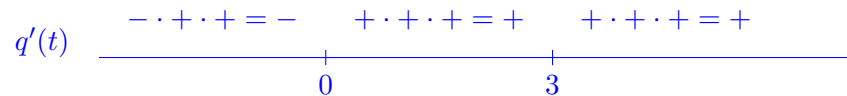
Throughout this problem, you must use calculus to find and justify your answers. Make sure you show enough evidence to justify your conclusions.

- a. [5 points] Find the  $t$ -coordinates of all local minimum(s) and local maximum(s) of  $q(t)$ . If there are none of a particular type, write NONE.

*Solution:* The critical points of  $q$  are where  $q'(t) = 0$ , which occurs at  $t = 0$  and  $t = 3$ . There are no points at which  $q'$  DNE.

(checking signs for 1st Derivative Test)	$t < 0$	$0 < x < 3$	$3 < t$
$t$	-	+	+
$e^{t/2}$	+	+	+
$ t - 3 $	+	+	+
$q'(t) = te^{t/2} t - 3 $	- · + · + = -	+ · + · + = +	+ · + · + = +

This gives the following number line for  $q'(t)$ :

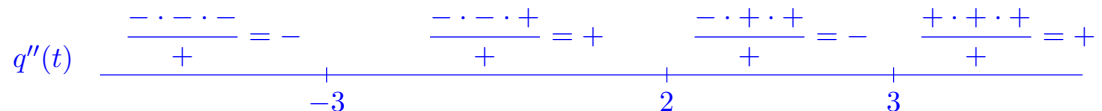


By the First Derivative Test,  $q(t)$  has a local min at  $t = 0$ . There is no local extremum at  $t = 3$ .

**Answer:** Local min(s) at  $t = \underline{0}$  and Local max(es) at  $t = \underline{\text{NONE}}$

- b. [4 points] Find the  $t$ -coordinates of all inflection points of  $q(t)$ , or write NONE if there are none.

*Solution:* We start by finding any values of  $t$  for which  $q''(t) = 0$  or  $q''$  DNE, and find  $t = -3$ , 2, and 3. Now we need to check to see whether the concavity of  $q$  changes at these points:



Because the sign of the second derivative of  $q$  changes at each of these points, they are all inflection points.

**Answer:** Inflection point(s) at  $t = \underline{-3, 2, 3}$