

3. [8 points] At a certain location in Lake Michigan, scientists are measuring water temperature. Let $W(d)$ be the temperature, in degrees Fahrenheit ($^{\circ}F$), of the water at a depth of d meters (m). Shown below is a table of values of $W(d)$ and its derivative $W'(d)$, which are both defined and differentiable for all $d \geq 0$.

d	10	18	20	36	78	95
$W(d)$	62	56	55	50	43	41
$W'(d)$	-1.25	-0.60	-0.45	-0.28	-0.15	-0.10

Assume that between each pair of consecutive values of d given in the table, each function $W(d)$ and $W'(d)$ is either always increasing or always decreasing. *Throughout this problem, you do not need to include units or simplify numerical values.*

- a. [1 point] Use the table to approximate the value of $W''(19)$.

Answer: $W''(19) \approx \underline{\underline{\frac{0.15}{2}}}$

- b. [2 points] Write a formula for the linear approximation $L(d)$ of $W(d)$ near $d = 95$.

Answer: $L(d) = \underline{\underline{41 - 0.1(d - 95)}}$

- c. [1 point] Use your formula from part b. to approximate the water temperature, in $^{\circ}F$, of the water at a depth of 90 meters.

Answer: $\underline{\underline{41 - 0.1(90 - 95) = 41.5}}$

- d. [1 point] Is your estimate from part c. an overestimate, an underestimate, neither, or is there not enough information to decide? Circle your answer.

Circle One: OVERESTIMATE UNDERESTIMATE NEITHER NOT ENOUGH INFO

- e. [3 points] The scientists are taking measurements using an underwater drone. The depth d , in meters, of the drone after t minutes of taking measurements can be modeled by $d = 3\sqrt{t}$. Let $R(t) = W(3\sqrt{t})$ be the temperature in $^{\circ}F$ outside the drone t minutes into the measurements. Write a formula for the linear approximation $K(t)$ of $R(t)$ near $t = 36$.

Solution: We know $K(t) = R(36) + R'(36)(t - 36)$. Now, $R(36) = W(3\sqrt{36}) = W(18) = 56$. Also, $R'(t) = W'(3\sqrt{t}) \cdot \frac{3}{2\sqrt{t}}$ by the Chain Rule, so $R'(36) = W'(18) \cdot \frac{3}{12} = -\frac{0.6}{4}$.

Answer: $K(t) = \underline{\underline{56 - \frac{0.6}{4}(t - 36)}}$