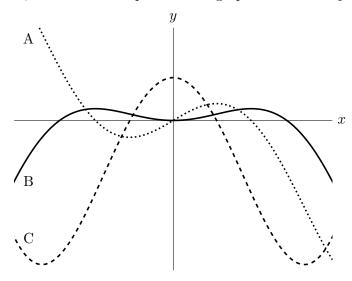
4. [4 points] Shown below are portions of the graphs of y = f(x), y = f'(x), and y = f''(x). Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



**Answer:** f(x): \_\_\_\_\_B

f'(x): A

 $f''(x) : \underline{C}$ 

**5.** [7 points] The function p(x) is given by the following formula, where c and d are nonzero constants:

$$p(x) = \begin{cases} \frac{1}{3}x^3 - 9x + 1 & x \le 0\\ 2^x & 0 < x < 2\\ c + d(x - 2) & x \ge 2. \end{cases}$$

**a.** [3 points] Find one pair of values for c and d such that p(x) is differentiable at x = 2. Show your work.

Solution: For p(x) to be differentiable at x=2, it must first be continuous there, so we need c+d(2-2)=c to equal  $2^2=4$ . Then, we need the slopes on either side of x=2 to be the same. For 0 < x < 2 we have  $p'(x) = \ln(2)2^x$ , while for x > 2 we have p'(x) = d, so we need  $d = \ln(2)2^2$ .

**Answer:**  $c = \underline{\hspace{1cm}} 4 \hspace{1cm} \text{and} \hspace{1cm} d = \underline{\hspace{1cm}} 4 \ln(2)$ 

**b.** [4 points] For the values of c and d from part **a.**, find the x-coordinates of all critical points of p(x) or write NONE if there are none. Show your work.

Solution: For x < 0 we have  $p'(x) = x^2 - 9$ . Then, as above, for  $0 < x \ge 2$  we have  $p'(x) = \ln(2)2^x$ , while for  $x \ge 2$  we have  $p'(x) = d = 4\ln(2)$ . Recall that we chose the values of c and d in part **a.** so that p(x) was differentiable at x = 2, so we know p'(2) exists. However, we need to consider whether p'(x) exists at x = 0: to the left of x = 0, the slope is  $0^2 - 9 = -9$ , while to the right, the slope is  $\ln(2)$ , and since these are not equal, p'(x) does not exist at x = 0, which is therefore a critical point of p(x).

Then, we note that p'(x) is never 0 for x > 0, but  $x^2 - 9 = 0$  when  $x = \pm 3$ . However, this formula is only relevant for x < 0, so this only gives -3 as an additional critical point.