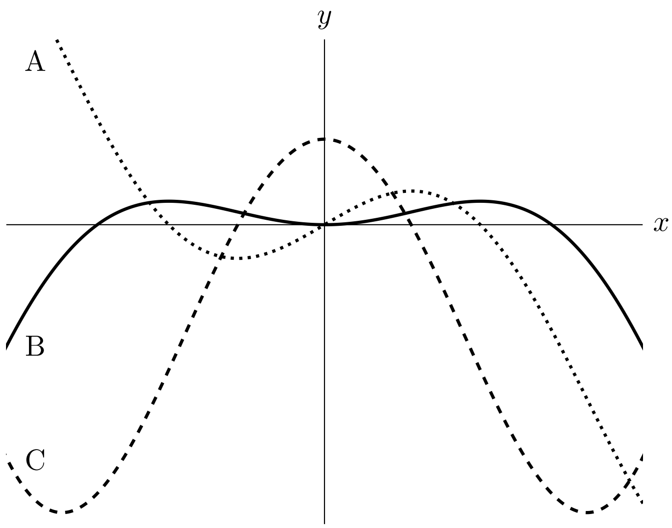
4. [4 points] Shown below are portions of the graphs of $y = f(x)$, $y = f'(x)$, and $y = f''(x)$. Determine which graph is which, and then, on the answer lines below, indicate after each function the letter A, B, or C that corresponds to its graph. No work or justification is needed.



Answer: $f(x)$: B

$f'(x)$: A

$f''(x)$: C

5. [7 points] The function $p(x)$ is given by the following formula, where c and d are nonzero constants:

$$p(x) = \begin{cases} \frac{1}{3}x^3 - 9x + 1 & x \le 0 \\ 2^x & 0 < x < 2 \\ c + d(x-2) & x \ge 2. \end{cases}$$

a. [3 points] Find one pair of values for c and d such that $p(x)$ is differentiable at $x = 2$. Show your work.

Solution: For $p(x)$ to be differentiable at $x = 2$, it must first be continuous there, so we need $c + d(2-2) = c$ to equal $2^2 = 4$. Then, we need the slopes on either side of $x = 2$ to be the same. For $0 < x < 2$ we have $p'(x) = \ln(2)2^x$, while for $x > 2$ we have $p'(x) = d$, so we need $d = \ln(2)2^2$.

Answer: $c =$ 4 and $d =$ $4\ln(2)$

b. [4 points] For the values of c and d from part **a.**, find the x-coordinates of all critical points of $p(x)$ or write NONE if there are none. Show your work.

Solution: For $x < 0$ we have $p'(x) = x^2 - 9$. Then, as above, for $0 < x \ge 2$ we have $p'(x) = \ln(2)2^x$, while for $x \ge 2$ we have $p'(x) = d = 4\ln(2)$. Recall that we chose the values of c and d in part **a.** so that $p(x)$ was differentiable at $x = 2$, so we know $p'(2)$ exists. However, we need to consider whether $p'(x)$ exists at $x = 0$: to the left of $x = 0$, the slope is $0^2 - 9 = -9$, while to the right, the slope is $\ln(2)$, and since these are not equal, $p'(x)$ does not exist at $x = 0$, which is therefore a critical point of $p(x)$.

Then, we note that $p'(x)$ is never 0 for $x > 0$, but $x^2 - 9 = 0$ when $x = \pm 3$. However, this formula is only relevant for $x < 0$, so this only gives -3 as an additional critical point.

Answer: Critical point(s) at $x =$ -3, 0

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